A Modulus Compensation Algorithm for Shape Self-Calibration of Paired Sensors Based Antennas

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Abstract—This paper is concerned with the array shape self-calibration problem when the array gain pattern of each sensor is spatially dependent and unknown. We adapt a Constant Modulus Approach (CMA) to improve the precision in the sensor localization. We will see how this original method conducts to build particular antenna configurations appropriate for self-calibration. The performance improvement lies in a strong bias reduction.

Keywords—Array Shape Self-Calibration, Constant Modulus Algorithm

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1. INTRODUCTION

Array processing algorithms for source localization can severely be degraded by uncertainties in the array shape. When flexible antenna arrays are used (consider for example an airborne antenna fitted under the wings), their actual shape can widely deviate from the nominal geometry. In such scenarios, the associated superresolution array processing dramatically fails, leading to high spatial ambiguity sidelobes.
It is well known that array shape self-calibration techniques can provide an estimation of sensor location using unknown narrow-band and far-field sources impinging on the array. Most of these methods suppose the array to be composed of isotropic sensors (see [1], [2], [4]). When the gain pattern of each sensor is spatially dependent and unknown, these approaches lead to biased estimated locations, this bias depending on the array deformation level.
In this paper, we propose to reduce significantly the sensor position estimation bias when the antenna can be considered as composed of several subarrays. Each subarray is made up of two sensors (with unknown but identical gain patterns) whose orientations are almost identical. Position and direction of the first subarray are assumed to be known. When the array is distorted, we reasonably assume that the deformation inside a subarray is negligible, and consequently the two sensors have the similar gains whatever the source bearing. Under this assumption, we develop a Modulus Compensation Algorithm (MCA) in order to estimate the position of each sensor.
N being the number of sources, the algebraical derivations conduct to the joint-diagonalization of N Hermitian matrices of rank 1 built from the Singular Value Decomposition of the spatial covariance matrix. We prove that the number of subarrays must be equal to or greater than $N^2 - N$ in order to satisfy the identifiability conditions.
Numerical simulations are provided in the two-dimensional case. Several configurations of antenna are compared. The environmental field involves sources with unknown parameters and additional Gaussian noise. Our approach is compared to the previous quoted algorithms and exhibits better performance.

2. BACKGROUND

We consider N distinct sources radiating unknown narrowband $s_j(t)$ centered at the same pulsation ω and received by a M-sensor antenna with unknown geometry. Localization of sources is unknown but assumed to be far enough from the array so that the signal wavefronts are planar. Moreover, we assume that the spatial diversity is full *i.e* wavefront normal vectors $\mathbf{n}_j$ are not two-by-two collinear. Finally and to simplify, we only consider the case where both the Directions Of Arrival (DOAs) $\mathbf{n}_j$ of the sources and the array are coplanar.

Source DOAs being unknown, "observability" of the sensor localization requires that at least the coordinates of one sensor and the direction of a second one are known. In this study, we

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will assume that the locations of the two first sensors, indexed by 1 and 2, are known. Thus, we set up a Cartesian coordinate system on the array plane with the origin at sensor 1. The coordinates of the location of sensor i relative to sensor 1 are denoted by (x_i, z_i) and sensor 2 have the coordinates $(x_2, 0)$. x_2 is assumed to be less than half of the sources wavelength λ corresponding to ω. The $M-2$ remaining elements can form, if needed, an incomplete array *i.e.* distances between two neighbor sensors may be greater than $\lambda/2$.

We assume that the array shape corresponds to a distorted version of *a priori* known nominal shape similar to a linear antenna shape (see fig. 1). The coordinates corresponding to the nominal sensor locations are denoted by (x_i^n, z_i^n).

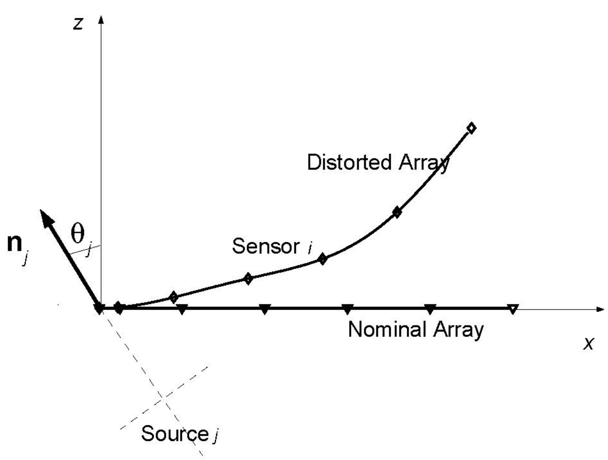

Figure 1. Problem Geometry

To avoid any ambiguity problem during the sensors localization, we only consider the case where, for each sensor, the distance between its distorted position and its nominal one does not exceed $\lambda/2$.

Classically, we define the M-component vector $\mathbf{a}_j$ to be the complex array response for the j^{th} source. A "snapshot" $\mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T$ taken by the sensors at time t can then be described by the matrix equation

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\eta}(t), \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_N(t)]^T$ is the source vector, $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N]$ represents the array response matrix and $\boldsymbol{\eta}(t) = [\eta_1(t), \eta_2(t), \ldots, \eta_M(t)]^T$ is the additive noise vector. The noise is assumed to have no statistical relation with the sources, to be spatially incoherent and to have the same power σ^2 on each sensor.

The $(M \times N)$ matrix $\mathbf{A} = [a_{ij}]$ is obviously unknown and accounts for gain and phase sensor deviations:

$$a_{ij} = g_{ij} \exp\{\mathrm{j}\phi_{ij}\}.$$

We consider only phase deviations due to the sensor positions (sensor gains are real) and consequently, the phase array response of the i^{th} sensor to a source j is

$$\phi_{ij} = \frac{2\pi}{\lambda}\mathbf{p}_i^T\mathbf{n}_j, \tag{2}$$

where $\mathbf{p}_i = [x_i, z_i]^T$ is the position vector of the i^{th} sensor. Since the vectors $\mathbf{n}_j$ are not collinear, note that matrix $\mathbf{A}$ is necessary full column rank. Still note that, since $\mathbf{p}_1 = [0\ 0]^T$, we have $a_{1j} = 1$ for all $j = 1\ldots, N$.

The array shape self-calibration problem is to estimate sensor locations $\mathbf{p}_i$ for all $i \in [3, \ldots, M]$ from the identification of the phases of the array response matrix $\mathbf{A}$ in an unknown source field.

3. Array Response Matrix Estimation Techniques

There are number of techniques available to identify the array response matrix. We choose to mention three of them:

In [1], Weiss and Friedlander propose to estimate alternatively the DOAs and the sensor locations until convergence is achieved. As in every iterative method, an ad-hoc initialization is necessary for sensor locations and for DOAs. The nominal array geometry is used together to initialize sensor locations and with MUSIC algorithm in order to provide initial DOAs. Note that MUSIC may have difficulties when incomplete arrays are used. This method requires that all the sensor gains g_{ij} are known.

With a Constant Modulus Algorithm [2], the same authors show that, when isotropic sensors are used, a direct estimation of the array response matrix is still possible avoiding thus any problem of initialization. If gains g_i (gains are no more dependent on the DOAs) are unknown, their estimation is possible assuming that the sources $s_j(t)$ are uncorrelated. For a given number of sources N, CMA needs at least $N^2 - N + 1$ sensors.

Assuming that the sources $s_j(t)$ are uncorrelated, the problem of the identification of the array response matrix can be linked to a Blind Source Separation (BSS) problem. For example, an algorithm such as AMUSE [3] can be used to estimate the matrix $\mathbf{A}$ if a sufficient number of snapshots $\mathbf{y}(t)$ is available and if sources are assumed to have different spectra. The main advantage of such BSS approaches is only N sensors are sufficient to deal with N sources.

When practical applications are considered, the previous methods fail for two main reasons. First, the sensor are rarely isotropic and even if their gain patterns are known, their spatial directions being, by assumption, unknown, the gains g_{ij} remains unknown too. Second, the number of snapshots is, in practice, insufficient to consider the different sources to be uncorrelated.

In this paper, we propose to adapt the Constant Modulus Algorithm (CMA) for accurate calibration when distorted non-isotropic sensor arrays are considered.

4. Modulus Compensation Algorithm (MCA)

From equation (1), the sampled data model becomes $\mathbf{y}(t_k) = \mathbf{A}\mathbf{s}(t_k) + \boldsymbol{\eta}(t_k), \quad k = 1, 2, \ldots, N_s$.

Let us introduce the $(M \times N_s)$ output matrix $\mathbf{Y} = [y_{ik}]$ with $y_{ik} = y_i(t_k)$, the $(N \times N_s)$ source matrix $\mathbf{S} = [s_{jk}]$ $(s_{jk} = s_j(t_k))$ and the $(M \times N_s)$ noise matrix $\boldsymbol{N} = [\eta_{ik}]$ $(\eta_{ik} = \eta_i(t_k))$. These three matrices together with matrix $\mathbf{A}$ obey the following equation

$$\mathbf{Y} = \mathbf{A}\mathbf{S} + \boldsymbol{N}. \tag{3}$$

The number of samples N_s is supposed to be large enough to ensure that the $(N \times N)$ matrix $\mathbf{S}\mathbf{S}^H$ is regular. Moreover, we assume that

1. $\frac{1}{N_s}\boldsymbol{N}\boldsymbol{N}^H$ is close to $\mathtt{Cov}\{\boldsymbol{\eta}\}$, say $\sigma^2\mathbf{I}_M$
2. $\frac{1}{N_s}\mathbf{S}\boldsymbol{N}^H$ is close to $\mathsf{E}\{\mathbf{S}\boldsymbol{N}^H\}$, the null matrix $\mathbb{O}_{NM}$

Consequently, in the sequel, we will consider that

- $\boldsymbol{N}\boldsymbol{N}^H \# N_s\sigma^2\mathbf{I}_M$
- $\mathbf{S}\boldsymbol{N}^H \# \mathbb{O}_{NM}$

Now, from (3) we have then

$$\mathbf{Y}\mathbf{Y}^H = \mathbf{A}\mathbf{S}\mathbf{S}^H\mathbf{A}^H + N_s\sigma^2\mathbf{I}_M. \tag{4}$$

We can then write the following eigenvalue decomposition

$$\mathbf{Y}\mathbf{Y}^H = \mathbf{U}\begin{bmatrix} \boldsymbol{\Lambda}_s & 0 \\ 0 & 0 \end{bmatrix}\mathbf{U}^H + N_s\sigma^2\mathbf{U}\mathbf{U}^H. \tag{5}$$

$\boldsymbol{\Lambda}_s + N_s\sigma^2\mathbf{I}_N$ is the diagonal matrix containing the N largest eigenvalues of $\mathbf{Y}\mathbf{Y}^H$. The first N columns of $\mathbf{U}$ correspond to the relevant unit-norm eigenvectors. The $M - N$ remaining vectors form an orthonormal basis for the $(M - N)$-dimensional orthogonal subspace.
Considering the expressions (4) and (5), $\mathbf{A}$ takes necessary the form

$$\mathbf{A} = \mathbf{U}_s\mathbf{W}, \tag{6}$$

where $\mathbf{U}_s$ contains the first N columns of $\mathbf{U}$ and where $\mathbf{W}$ is an unknown $N \times N$ matrix we have now to find in order to estimate $\mathbf{A}$.

An entry a_{ij} of $\mathbf{A}$ can be written $a_{ij} = \mathbf{u}_i^H\mathbf{w}_j$, where $\mathbf{u}_i^H$ is the i^{th} row of $\mathbf{U}_s$ and $\mathbf{w}_j$ is the j^{th} column of $\mathbf{W}$. Any column vector $\mathbf{w}_j$ must satisfy the equation

$$\mathbf{w}_j^H\mathbf{u}_i\mathbf{u}_i^H\mathbf{w}_j = g_{ij}^2 \quad \forall i, j. \tag{7}$$

Now, remember that we deal with an antenna whose shape corresponds to a distorted version of a linear one. Let us assume that the sensors have almost the same gain patterns and had initially (for the nominal shape) close directions. When the antenna is distorted, it is reasonable to think that, two neighbor sensors keep almost the same gain.

Ideally, assume that the array is composed of K couples of sensors (k_i, l_i) such as, for all $i = 1, \ldots, K$, we have

$$\forall j, \quad g_{k_i,j} = g_{l_i,j}. \tag{8}$$

This will be, for example, the case in practice, when the sensors of a same subarray are mounted on a rigid mechanical structure. Note that two couples of sensors must be independent (*i.e.* they must not share a same sensor). These different sensor couples are called subarrays.

From the different rows of $\mathbf{U}$, we can form the K matrices $\mathbf{Q}_i \triangleq \mathbf{u}_{k_i}\mathbf{u}_{k_i}^H - \mathbf{u}_{l_i}\mathbf{u}_{l_i}^H, \quad i = 1, \ldots, K$. Now, considering the equations (7) and (8) one has

$$\mathbf{w}_j^H\mathbf{Q}_i\mathbf{w}_j = 0 \quad \forall i, j. \tag{9}$$

Let us introduce the $\mathtt{vec}\{.\}$ operator that vectorizes a matrix by stacking its columns. There is a useful relationship between the Kronecker product $\otimes$ and the $\mathtt{vec}\{.\}$ operator:

$$\mathtt{vec}\{\mathbf{A}\mathbf{B}\mathbf{C}\} = (\mathbf{C}^T \otimes \mathbf{A})\mathtt{vec}\{\mathbf{B}\}.$$

Thus, vectorization of (9) yields

$$(\mathbf{w}_j^T \otimes \mathbf{w}_j^H)\mathtt{vec}\{\mathbf{Q}_i\} = \mathtt{vec}^T\{\mathbf{Q}_i\}(\mathbf{w}_j \otimes \mathbf{w}_j^*) = 0 \quad \forall i, j.$$

Using a matrix notation, this set of equations amounts to

$$\mathbf{Q}(\mathbf{w}_j \otimes \mathbf{w}_j^*) = \mathbf{0} \quad \forall j, \tag{10}$$

where the i^{th} row of the $K \times N^2$ matrix $\mathbf{Q}$ is $\mathtt{vec}^T\{\mathbf{Q}_i\}$. Any column vector $\mathbf{w}$, in $\mathbf{W}$, is such that $(\mathbf{w} \otimes \mathbf{w}^*)$ is in the null space of $\mathbf{Q}$ denoted by $\mathcal{N}\{\mathbf{Q}\}$.
Now, we can exhibit the following sufficient condition for $\mathbf{A}$ to be unique:

Proposition 1: A full column rank matrix $\mathbf{A}$ satisfying the equation (6) under constraint (10) is unique up to a permutation and a scaling factor over its columns if $\dim\mathcal{N}\{\mathbf{Q}\} = N$.

Proof: Since the matrix $\mathbf{A}$ is full column rank and according to equation (6), the vectors $\mathbf{w}_j \otimes \mathbf{w}_j^*, j \in \{1, \ldots, N\}$ are linearly independent and thus form a basis of a N-dimensional subspace. If, by assumption, $\dim \mathcal{N}\{\mathbf{Q}\} = N$ then, according to (10), this subspace is necessary $\mathcal{N}\{\mathbf{Q}\}$.
Let us now establish that this basis $\{\mathbf{w}_j \otimes \mathbf{w}_j^*\}$, $j \in \{1, \ldots, N\}$ is unique.

In the subspace $\mathcal{N}\{\mathbf{Q}\}$, any vector in the form $\mathbf{v} \otimes \mathbf{v}^*$ can be written as:

$$\mathbf{v} \otimes \mathbf{v}^* = \sum_{j=1}^{N} \gamma_i \mathbf{w}_j \otimes \mathbf{w}_j^*.$$

We can equivalently rearrange the previous vector decomposition in terms of rank-1 matrix decomposition using the $\mathsf{vec}^{-1}\{.\}$ operator:

$$\mathsf{vec}^{-1}\{\mathbf{v} \otimes \mathbf{v}^*\} = \mathbf{v}^* \mathbf{v}^T = \sum_{j=1}^{N} \gamma_j \mathbf{w}_j^* \mathbf{w}_j^T.$$

Since all the matrices involved are rank-1 matrices, it is straightforward that coefficients α_j have to be all zero except for one: any vector structured as $\mathbf{v} \otimes \mathbf{v}^* \in \mathcal{N}\{\mathbf{Q}\}$ is collinear to one of the vectors $\mathbf{w}_j \otimes \mathbf{w}_j^*$. We can then assert that $\mathbf{W}$ (and consequently $\mathbf{A}$) is unique except for one permutation and a scaling factor over its columns.

■

In the sequel, we assume that $\dim \mathcal{N}\{\mathbf{Q}\} = N$. Validity of this assumption needs a sufficient spatial diversity; this point will not be discussed here. Having said that, a necessary condition is that matrix $\mathbf{Q}$ contains $N^2 - N$ independent rows. In other words, the array must be composed of, at least $(N^2 - N)$ independent sensor couples *i.e.* $2(N^2 - N)$ sensors.

Given $\{\mathbf{b}_1, \ldots, \mathbf{b}_N\}$ an arbitrary basis of $\mathcal{N}\{\mathbf{Q}\}$, one can express each vector $\mathbf{b}_k$ as,

$$\mathbf{b}_k = \sum_{j=1}^{N} \alpha_j^k \mathbf{w}_j \otimes \mathbf{w}_j^*, \forall k \in \{1, 2, \ldots, N\}.$$

Performing the inverse vec operation on previous equation we get

$$\mathsf{vec}^{-1}\{\mathbf{b}_k\} = \sum_{j=1}^{N} \alpha_j^k \mathbf{w}_j^* \mathbf{w}_j^T = \mathbf{W}^* \mathbf{\Sigma}_k \mathbf{W}^T, \forall k \in \{1, \ldots, N\},$$

where $\mathbf{\Sigma}_k = \mathsf{diag}\{\alpha_1^k, \ldots, \alpha_N^k\}$.

The problem of the identification of $\mathbf{W}$ merge then with the diagonalization of $\mathbf{R}_1^{-1}\mathbf{R}_2$ where $\mathbf{R}_1$ and $\mathbf{R}_2$ are two matrices among the set of matrices $\mathbf{R}_k \triangleq \mathsf{vec}^{-1}\{\mathbf{b}_k\}$ $k = 1, \ldots, N$.
The eigenvectors matrix of $\mathbf{R}_1^{-1}\mathbf{R}_2$ straightforward corresponds to the inverse of the transposed matrix $\mathbf{W}$ we are looking for, except for an unknown permutation and an unknow complex scaling factor over its columns.

To replace $\mathbf{W}$ by such a matrix in equation (6) conducts to an estimation $\hat{\mathbf{A}}$ of the array response matrix $\mathbf{A}$. Each column vector in $\hat{\mathbf{A}}$ have to be divided with its first component in order to eliminate the unknown scaling factors and to ensure that $\hat{a}_{1j} = 1$ for all j.
Note that, in presence of noise, more robust methods based on joint-diagonalization of the whole matrices $\mathbf{R}_k$, $k = 1, \ldots, N$ are preferable (see [5], [6] for deta ils).

5. Sensor Localization Procedure

In this section, we assume that an estimation $\hat{\mathbf{A}}$ of the array response matrix $\mathbf{A}$ was provided by the MCA algorithm.

DOA Estimation

Since the positions of the two first sensors are the only known, DOAs can be estimated using these sensors only. In $\hat{\mathbf{A}}$, the quantity $\angle \hat{a}_{ij}$ correspond to an estimation of the relative phase for the source j between the sensor i and the reference sensor 1. Recall that the argument of a complex number is always given in the interval $[-\pi, \pi]$ and generally, we have according to (2)

$$\begin{aligned} \angle \hat{a}_{ij} &\equiv \phi_{ij} \mod [2\pi] \\ &\equiv \frac{2\pi}{\lambda} \mathbf{n}_j^T \mathbf{p}_i \mod [2\pi] \end{aligned}$$

The distance between the sensors 1 and 2 being, by assumption lower than $\lambda/2$, the phases ϕ_{2j} are necessary in the interval $[-\pi, \pi]$ and we have

$$\angle \hat{a}_{2j} = \phi_{2j} = \frac{2\pi}{\lambda} \mathbf{n}_j^T \mathbf{p}_2 \quad \forall j.$$

With $\mathbf{n}_j = [\sin\theta_j, \cos\theta_j]^T$, the DOAs θ_j are then estimated using

$$\hat{\theta}_j = \mathsf{asin} \frac{\lambda}{2\pi} \frac{\angle \hat{a}_{2j}}{\|\mathbf{p}_2\|}, \quad \forall j = 1, \ldots, N. \tag{11}$$

Sensor Position Estimation

To avoid any problem of missing rotations in the estimation of the phases ϕ_{ij} it is convenient to use the notion of array nominal shape.
Given the nominal position vectors $\mathbf{p}_i^n$ and the estimated DOAs $\hat{\theta}_j$, we can compute the array response matrix $\mathbf{A}^n$ for the nominal array. We can still compute a matrix $\mathbf{A}'$ whose entries are obtained dividing each entry in $\hat{\mathbf{A}}$ with its corresponding entry in $\mathbf{A}^n$. Arguments $\angle \hat{a}'_{ij}$ of the elements of $\mathbf{A}'$

represent the relative phases between actual and nominal sensor locations. Now, if we assume that the nominal sensor locations and the corresponding actual locations are lower than $\lambda/2$, then these relative phases are necessary in the interval $[-\pi, \pi]$ and can be straightforward estimated from the angles of $\mathbf{A}'$.

Introducing $\mathbf{p}'_i \triangleq \mathbf{p}_i - \mathbf{p}^n_i, \quad i = 1, \ldots, M$, the relative sensor locations, we have then the following relations

$$\angle \hat{a}'_{ij} = \frac{2\pi}{\lambda} \mathbf{n}_j^T \mathbf{p}'_i \quad \forall i, j.$$

An estimation of $\mathbf{p}'_i$ in the least square sense can then be obtained using

$$\mathbf{p}'_i = \frac{\lambda}{2\pi} \begin{bmatrix} \hat{\mathbf{n}}_1^T \\ \vdots \\ \hat{\mathbf{n}}_N^T \end{bmatrix}^{\dagger} \begin{bmatrix} \angle \hat{a}'_{i1} \\ \vdots \\ \angle \hat{a}'_{iN} \end{bmatrix}, j = 1, \ldots, M,$$

where $\hat{\mathbf{n}}_j = [\sin \hat{\theta}_j, \cos \hat{\theta}_j]^T, \quad j = 1, \ldots, N$ is the estimated wavefront normal vectors and where the upperscript $\dagger$ denotes the Moore-Penrose pseudoinverse.

6. Numerical Examples

In this section, we present the results of three antenna configurations that demonstrate the performance of the proposed method. In all experiments, the sensors used have all the same real gain pattern $G(\theta)$

$$\begin{cases} G(\theta) = \mathbf{n}_s^H \mathbf{n}_w, \text{ if } \theta \in [-\pi/2, \pi/2], \\ G(\theta) = 0, \text{ otherwise.} \end{cases}$$

$\mathbf{n}_s$ is a unit-norm vector giving the direction of the sensor and where $\mathbf{n}_w$ is a unit-norm vector giving the direction of the wave (see fig. 2).

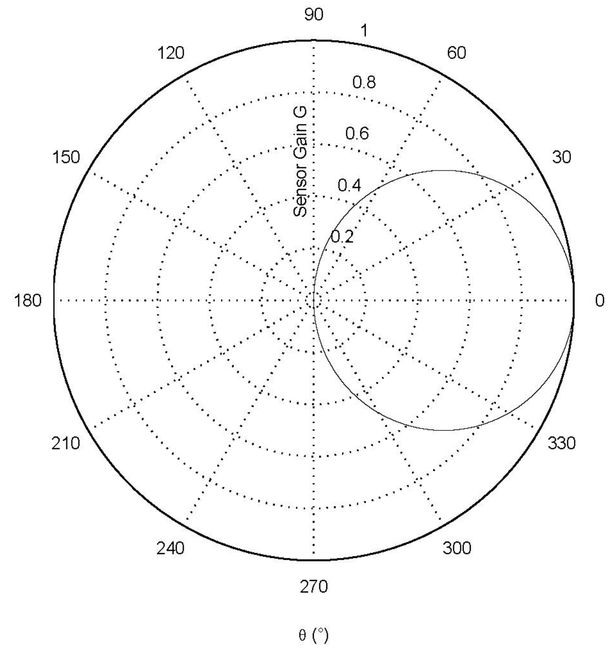

Figure 2. Sensor Gain Pattern

In all the simulations, we consider 3 narrowband equal-power source signals generating a wave of $\lambda = 30$cm located in the far field of the array at directions $\theta_1 = -36^o$, $\theta_2 = -3^o$ and $\theta_3 = 20^o$.
These 3 sources are differently colored and impinge upon a 18-element incomplete arrays whose the two first elements are $0.8\lambda/2$ apart.
The recordings are corrupted by spatially incoherent Gaussian white noises.
These three antennas correspond to distorted versions of a linear ones; all are about 4m long. When the array is distorted, the terminal sensor is subjected to a 90 percent of $\lambda/2$ shift.
The notion of subarrays, appropriate when MCA is used, fully appears in the two last antenna configurations.

Simulations use $N_s = 10000$ samples and the sensor noise is injected with an SNR of 30dB on each sensor. 30 independent simulation runs are used to compare the self-calibration results obtained with MCA, CMA and AMUSE algorithms.

Antenna Configuration 1

When at rest (nominal shape), this first antenna form, starting from the third sensor, a regular incomplete array whose sensors are 0.8λ apart (see fig. 3). Here, MCA is applied considering that the 9 sensor couples $(2p - 1, 2p), \quad p = 1, \ldots, 9$ form as many subarrays.

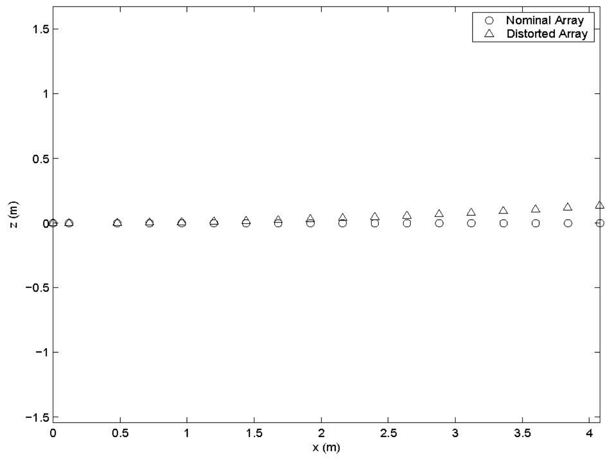

Figure 3. Antenna 1

Fig. 4 depicts terminal sensor location estimations obtained with MCA and CMA. One can see that MCA have a lower average location error (bias) than CMA. Unfortunately, it have to trade standard deviation off against bias.

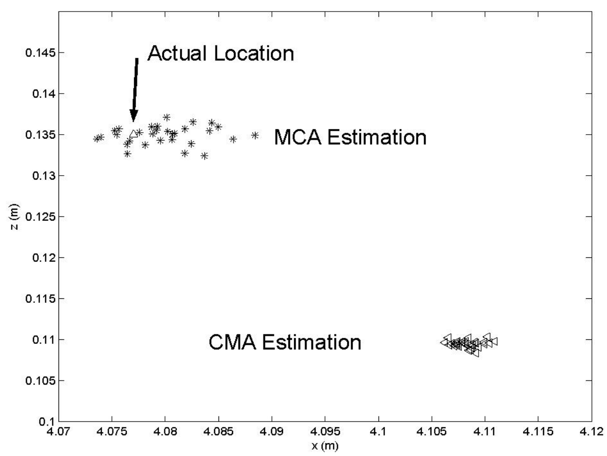

Figure 4. Terminal Sensor Localization

The following tabular exposes, always for the localization of the last sensor, the performances of the different algorithms. AMUSE bad performances are due solely to the fact that the number of samples is insufficient to consider the different sources to be uncorrelated.

	Bias (mm)		STD (mm)	
	x	z	x	z
MCA	2.9	-0.1	3.7	1.1
CMA	31.3	-25.5	1.2	0.4
AMUSE	-25.4	-32.2	85.6	45.5

Table 1. Antenna 1 Results

Antenna Configuration 2

This second antenna have the same dimension than the previous one but the sensors have been brought closer in order to form 1.6λ apart couples (subarrays). Inside a subarray, the two sensors are $0.8\lambda/2$ apart (see fig. 5). Direction of each sensor depends on the curvature of the array and consequently, even inside a subarray, the sensors don't focus toward the same direction.

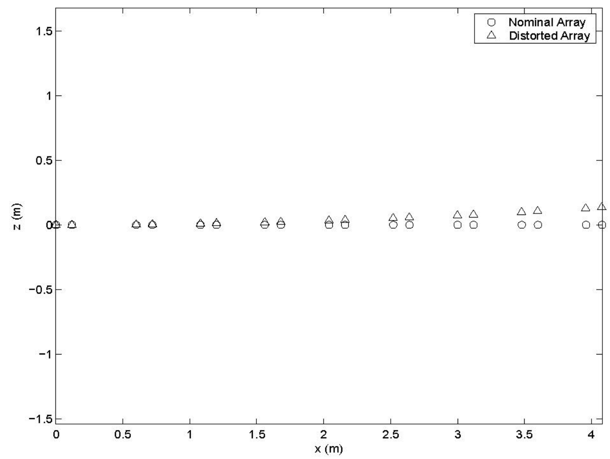

Figure 5. Antenna 2

As previsously, fig. 6 presents terminal sensor location estimations obtained with MCA and CMA. Applying MCA with this subarray-based antenna conducts to a very lower average sensor location error (compared with CMA) but always an higher standard deviation.

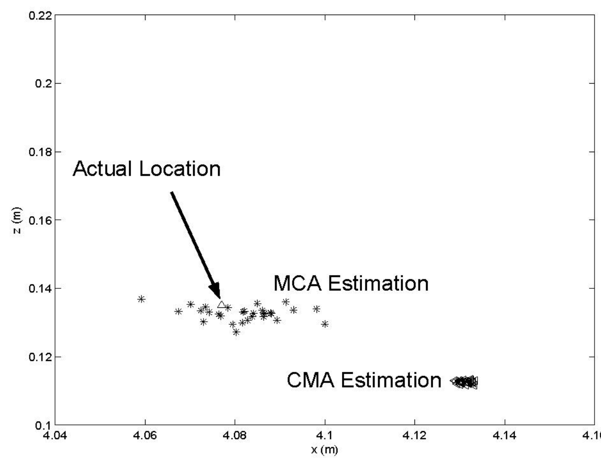

Figure 6. Terminal Sensor Localization

We resume in the following tabular the performances of the different algorithms for the localization of the last sensor. Note that AMUSE provide quite the same results than in the previous configuration antenna. The performances of AMUSE depend only on the sample number.

	Bias (mm)		STD (mm)	
	x	z	x	z
MCA	4.8	-2.3	8.8	2.1
CMA	54.2	-22.3	1.3	0.4
AMUSE	-25.2	-33.1	85.3	42.5

Table 2. Antenna 2 Results

Antenna Configuration 3

This last antenna is composed with the same subarrays as in the antenna configuration 2 but inside a subarray, the two sensors have exactly the same direction (undeformable subarrays).

Once again fig. 7 and table 3 show terminal sensor location estimations obtained with MCA and CMA. Once again we observe the same performance exchange between bias and variance.

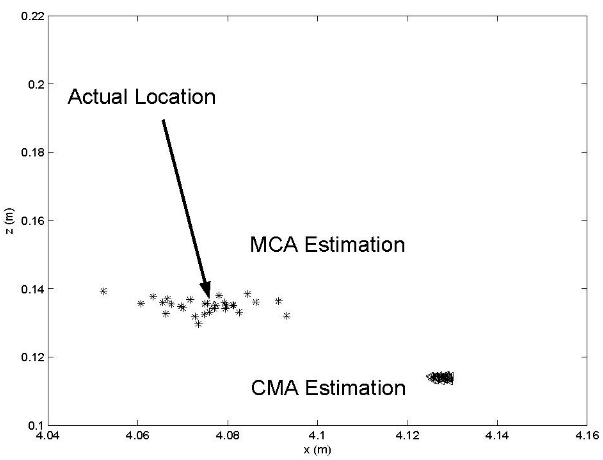

Figure 7. Terminal Sensor Localization

	Bias (mm)		STD (mm)	
	x	z	x	z
MCA	-1.9	0.1	8.8	2.1
CMA	54.6	-24.4	1.3	0.4
AMUSE	-26.2	-33.2	82.6	41.5

Table 3. Antenna 3 Results

7. Concluding Remarks

In this paper, we showed how to improve the sensor localization precision modifying the classical Constant Modulus Approach when the antenna is composed of non-isotropic and non identically oriented sensors.
The main result of the method is a strong reduction of the bias in the array shape estimation compared to other methods.
The main disadvantage is that compared to CMA in with an isotropic-sensor based antenna, the number of sources we are able to deal with, is lower.
Examples of antenna configurations appropriate for our method have been successfully tested out.

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