Array Shape Self-Calibration for Large Flexible Antenna

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Abstract—This paper presents a method dealing with the self-calibration problem in term of array shape for large flexible antenna. This implies to take into account the phenomena of distortion and vibration that this kind of antenna, like an array mounted under flexible wing, can suffer from. We propose a technique that eliminates, on the first part, the phase ambiguities due to the large static bending, and on the second part, that estimates an instantaneous array shape. This two-step method allows us to follow the antenna during its dynamic fluctuations due to vibrating modes. We present simulation results in case of two particular large flexible antennas.

Keywords—Target detection, adaptive tests, sequential detection.

TABLE OF CONTENTS

1. INTRODUCTION

Direction finding of emitting sources using sensor array requires the knowledge of sensor locations. An high angular accuracy needs the use of large-size sensor arrays. Technically, such antennas are often integrated to structures exposed to significant unknown distortions.
Using unknown impinging signal sources (for example TV or radio emissions...), allows one to estimate the sensor locations. In the past, Rockah and Schultheiss [1] are one of the first to assess the validity of self-calibration principle. They have shown that three sources are necessary for calibrate the array providing we know the exact position of one sensor and the direction of an other one relatively to the first one. Later on, Weiss and Friedlander presented a self-calibration technique based on maximum likelihood theory [2]. This iterative method juggle between two steps of parameters estimation : DOA and coordinates. As all iterative algorithms, we need an initialization processing, here we use the MUSIC algorithm using the initial known sensor positions. Then, Weiss and Friedlander and others authors, develop direct methods [3]. All parameters are estimated at the same time. Flanagan and Bell suggest a method for high static distortion magnitude [4]. All these methods give good results providing the sensor deformations are small, and more precisely, smaller than the wavelength. The classical self-calibration methods succeed provided that first, the antenna is subject to static deformation only and second, this remains close to an a priori known nominal shape.

This paper presents a method dealing with array-shape self-calibration in large dynamic deformation cases encountered in airborne antennas where sensors are mounted under flexible wings. In this case the shape of the antenna is the superposition of a large static bending (compared to the at rest shape) and low magnitude dynamic fluctuations due to vibrating modes. Such antennas reach up more than 10 meters in length and the maximum deformation can exceed 1m at the wing tip. On the other hand, we can assume that deformations near the fuselage are negligible. Generally, only the very low frequency vibration modes (around 10Hz) are significant, whose magnitudes are roughly 10% of the maximum static deformation in steady flight.

The sensors are assumed omnidirectional. This paper is organized as follows: in section 2, we specify the problem and the data model encountered in such antennas.
Then, in section 3, we detail the Constant Modulus Approach (CMA) [3], we use in a simplified case of data model (the free noise case) and we detail also an original technique in order to eliminate the phase ambiguities due to the static bending.

The central part of the paper is the section 4, where we detail the two steps method proposed for self-calibration in presence of noise. The aim of the first initialization step is to counteract the phase ambiguities by crosschecking several estimations provided by different subsets of source signals.

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This step being unfortunately noise sensitive, a significant observation time is necessary. We obtain only a coarse shape close to the static one. The second step is used to obtain the instantaneous array shape from the estimated static one. The problem reduces here to a classical self-calibration problem. The two steps are based on the Constant Modulus Approach.

Finally, the algorithm is tested on simulated data in the two-dimensional case, and its performance are compared to the Cramer Rao Bounds.

2. Problem Formulation and Data Model

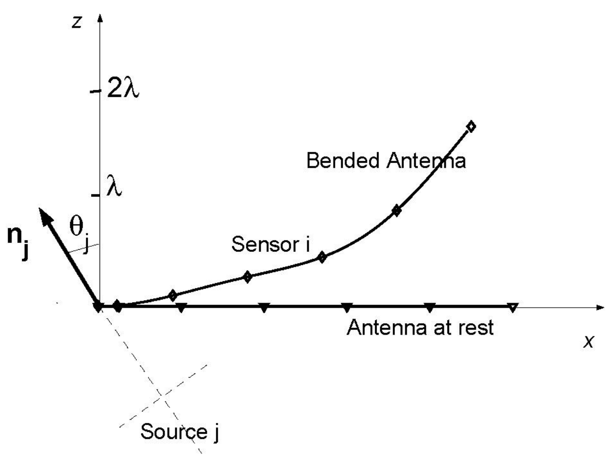

Figure 1. Antenna Geometry.

Our aim is to estimate the positions of M sensors of a dynamically bended antenna from N ($M > N$) narrowband radiating sources. In this study we assume that the antenna and sources are coplanar. The current position vector of the i^{th} sensor is then denoted $\mathbf{p}_i(t) = [x_i(t), z_i(t)]^T$. The positions of the first two sensors are known and the distance between them is lower or equal than half the wavelength λ corresponding to the carrier frequency of all the sources. The origin of the plane space is given by the position of the first sensor, and the position of the second sensor defines the direction of x-axis (see fig.1). Moreover, nominal sensor locations (for example corresponding to the antenna at rest) are known.
The inter-sensor distance except the distance between the two first sensors can be greater than $\lambda/2$. The same way, the distance between the current location and at rest location of a sensor can be greater than $\lambda/2$. This justifies the expression "large antennas" and "large deformations".

The Direction Of Arrival of each source is the same for all sensors (far field assumption) and differs from one source to another. With the axis system described in fig.1, the DOA of the j^{th} source is given by the unit vector $\mathbf{n}_j = [\sin(\theta_j), \cos(\theta_j)]^T$. All sources are in the same half-plane: $-\pi/2 < \theta_j < \pi/2$ for all $j \in \{1, \ldots, N\}$. We assume there is no signal reflection (singlepath propagation assumption).

Considering baseband signal, an output can be expressed at each time as a complex linear combination of the N demodulated sources. Denoting $\mathbf{s}(t) = [s_1(t), \ldots, s_N(t)]^T$ the vector of the N baseband sources, the signal at the output of the sensors can be described by the M-dimensional vector $\mathbf{y}(t) = [y_1(t), \ldots, y_N(t)]^T$:

$$\mathbf{y}(t) = \mathbf{A}(t)\mathbf{s}(t) + \boldsymbol{\eta}(t), \tag{1}$$

where $\boldsymbol{\eta}(t)$ is the noise vector and $\mathbf{A}(t)$, the array response matrix. The columns of the matrix $\mathbf{A}(t)$ are the steering vectors of each source.
We assume that we use omnidirectional identical sensors: the power of the contribution of a source signal on a sensor is the same for any sensor and any orientation of sensor. It will be taken equal to g, an unknown constant.
Under these assumptions, an entry $a_{ij}(t)$ of the previous matrix can be expressed only from the current sensor i location and the DOA $\mathbf{n}_j$ as

$$a_{ij}(t) = \exp\left\{ \mathrm{j}\frac{2\pi}{\lambda}\mathbf{p}_i^T(t)\mathbf{n}_j \right\}. \tag{2}$$

The array shape self-calibration problem is to estimate, at time t, the sensor locations $\mathbf{p}_i(t)$ for all $i \in [3, \ldots, M]$ from identification of the phase of the array response matrix $\mathbf{A}(t)$.

3. Array Self-Calibration in the Free Noise Case: A Direct Method for Large Deformations

Array Response Matrix identification

The following derivations are strongly inspired by the Constant Modulus Algorithm developed by Weiss and Friedlander, see [3].

From equation (1), the sampled data model in the free noise case becomes $\mathbf{y}(t_k) = \mathbf{A}(t_k)\mathbf{s}(t_k), \quad k = 1, 2, \ldots, N_S$.

Time Independence Assumption: We assume that the sampling frequency is chosen sufficiently high and the sample number N_S sufficiently small such that the matrices $\mathbf{A}(t_k)$ are locally time independent, so equal to a matrix $\mathbf{A}$ for the times $t_k, \quad k = 1, \ldots, N_S$.

For example if we consider $Ns = 6$ (that is a sufficient number of samples in a free noise case and for a number of sources $N < 6$), a sampling frequency = 60 kHz, and a mechanical vibration frequency = 10Hz, the observation duration will be equal to 0.05 % of the mechanical vibration period.

Introducing the output matrix $[\mathbf{Y}_{ik}] = [y_i(k)]$, ($i = 1, \ldots, M$), ($k = 1, \ldots, N_S$) and the ($N \times N_S$) source matrix $[\mathbf{S}_{jk}] = [s_j(k)]$) the data model becomes

$$\mathbf{Y} = \mathbf{A}\mathbf{S} \tag{3}$$

Provided that the number of samples is greater than the number of sources ($N_s \geq N$), and that the sources are non-correlated, $\mathbf{S}$ is full rank.
From (3) we have:

$$\mathbf{YY}^H = \mathbf{ASS}^H\mathbf{A}^H.$$

Consider the Eigenvalue Decomposition of the matrix

$$\mathbf{YY}^H = \mathbf{U\Lambda U}^H$$

where $\mathbf{\Lambda}$ is the diagonal matrix of the N non-null eigenvalues of $\mathbf{YY}^H$, and $\mathbf{U}$ is the $M \times N$ matrix of relevant eigenvectors.
Considering the two previous expressions for the matrix $\mathbf{YY}^H$, $\mathbf{A}$ takes the form

$$\mathbf{A} = \mathbf{UW}. \tag{4}$$

The problem is now reduced to the estimation of the matrix $\mathbf{W}$.

An entry a_{ij} of $\mathbf{A}$ can be written $a_{ij} = u_i^H w_j$, where u_i^H is the i^{th} row of $\mathbf{U}$ and w_j is the j^{th} column of $\mathbf{W}$. Each entry of a_{ij} having the same modulus, we can write

$$w_j^H u_i u_i^H w_j = g \quad \forall i, j \tag{5}$$

From the different rows of $\mathbf{U}$ we form the $M-1$ matrices $\boldsymbol{K}_i = u_{i+1}u_{i+1}^H - u_1 u_1^H \quad i = 1, \ldots, M-1$. Considering the equation (5) we get

$$w_j^H \boldsymbol{K}_i w_j = 0 \quad \forall i, j \tag{6}$$

Using the property $\mathsf{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathsf{vec}(\mathbf{B})$, vectorizing the previous equation yields

$$(w_j^T \otimes w_j^H)\mathsf{vec}(\boldsymbol{K}_i) = \mathsf{vec}^T(\boldsymbol{K}_i)(w_j \otimes w_j^*) = 0 \quad \forall i, j$$

This set of equation amounts to

$$\mathbf{K}(w_j \otimes w_j^*) = 0 \quad \forall j$$

where the i^{th} row of $\mathbf{K}$ is given by $\mathsf{vec}^T(\boldsymbol{K}_i)$.
The N vectors $(w_j \otimes w_j^*)$ are in the kernel of $\mathbf{K}$ whose dimension is $((M-1) \times N^2)$.

A necessary condition for the kernel of $\mathbf{K}$ to be spanned by the N vectors $(w_j \otimes w_j^*)$ is that the rank of $\mathbf{K}$ is equal to $N^2 - N$. Thus, we need at least $N^2 - N$ linearly independent vectors $\mathsf{vec}^T(\boldsymbol{K}_i)$ that means we need at least $N^2 - N + 1$ sensors.
Given $\{\mathbf{b}_1, \ldots, \mathbf{b}_N\}$ a basis of the kernel of $\mathbf{K}$, one can express each basis vector as

$$\mathbf{b}_k = \sum_{j=1}^{N} \alpha_j (w_j \otimes w_j^*) \quad \forall k \in \{1, 2, \ldots, N\}.$$

Performing the vec^{-1} operator on previous equation we get

$$\mathsf{vec}^{-1}(\mathbf{b}_k) = \sum_{j=1}^{N} \alpha_j (w_j^* w_j^T) = \mathbf{W}^* \mathbf{\Sigma_k} \mathbf{W}^T \, \forall k \in \{1, \ldots, N\},$$

where $\mathbf{\Sigma_k} = \mathsf{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_N\}$.

The problem of the identification of $\mathbf{W}$ can merge with diagonalization of $\boldsymbol{R}_1^{-1}\boldsymbol{R}_2$ where $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are two matrices among the set of matrices $\boldsymbol{R}_k = \mathsf{vec}^{-1}(\mathbf{b}_k) \quad k = 1, \ldots, N$.

$$\boldsymbol{R}_1^{-1}\boldsymbol{R}_2 = \mathbf{W}^{-T}\mathbf{\Sigma}\mathbf{W}^T \tag{7}$$

where $\mathbf{\Sigma} = \mathbf{\Sigma}_1^{-1}\mathbf{\Sigma}_2$ is diagonal.
Except for a permutation and a complex factor, the matrix of eigen vectors of $\boldsymbol{R}_1^{-1}\boldsymbol{R}_2$ is the inverse of the tranposed matrix $\mathbf{W}$ we are looking for. Replacing $\mathbf{W}$ by such a matrix in equation (4), and normalizing each column of the matrix result obtained by the first term of each column we obtain the estimated matrix $\hat{\mathbf{A}}$, with $\hat{a}_{1j} = 1 \forall j$ and $\hat{a}_{ij} = \exp(\mathrm{j}2\pi\phi_{\mathrm{ij}})$.

Note that in presence of noise, more robust methods based on joint diagonalization of all matrices will be chosen.

Sensor localization - Phase ambiguity elimination

We consider ϕ_{ij} the relative phase for the source j between the sensor i and the reference sensor 1. It is given in $[-\pi, \pi]$ and it is proportional to the projection of the position vector of sensor i on the DOA unit vector j modulo 2π (see equation (2)):

$$\phi_{ij} \equiv \frac{2\pi}{\lambda}\mathbf{p}_i^T\mathbf{n}_j \mod [2\pi]$$

Let us recall that the norm of sensor 2 position vector is known and lower than $\lambda/2$. Moreover the reference axes are chosen such as $\mathbf{p}_2 = [\|\mathbf{p}_2\|, 0]^T$ and $\mathbf{n}_j = [\sin(\theta_j), \cos(\theta_j)]^T$.
From the previous equation the DOAs are computed as

$$\theta_j = \operatorname{asin}\left(\frac{\phi_{i2}\lambda}{2\pi\|\mathbf{p}_2\|}\right), \tag{8}$$

Given the at rest position vectors $\mathbf{p}_i^r$, the unknowns become the difference vector between the at rest and the actual vector

$$\mathbf{p}_i' = \mathbf{p}_i - \mathbf{p}_i^r \quad i > 2. \tag{9}$$

We introduce ϕ_{ij}' the relative phase between actual and at rest sensor i:

$$\phi_{ij}' \equiv \frac{2\pi}{\lambda}\mathbf{p}_i'^T\mathbf{n}_j \mod [2\pi]$$

This relative phase is deduced from the measured phase ϕ_{ij} by

$$\phi_{ij}' = \phi_{ij} - \frac{2\pi}{\lambda}\mathbf{p}_i^{\mathbf{r}T}\mathbf{n}_j, \quad \forall i > 2,$$

hence

$$\mathbf{n}_j^T \mathbf{p}_i' = \frac{\lambda}{2\pi}\left(\phi_{ij}' + k_{ij}2\pi\right) \quad i > 2, \quad k_{ij} \in \mathbb{Z}.$$

For a sensor i, the relative position vector is obtained from a number of sources greater or equal than the space dimension of the problem:

$$\mathbf{p}_i'(\mathbf{k}_i) = \frac{\lambda}{2\pi}\begin{bmatrix} \mathbf{n}_1^T \\ \vdots \\ \mathbf{n}_N^T \end{bmatrix}^\sharp \begin{bmatrix} \phi_{i1}' + k_{i1}2\pi \\ \vdots \\ \phi_{iN}' + k_{iN}2\pi \end{bmatrix} \tag{10}$$

where the superscript $\sharp$ denotes the Moore-Penrose pseudo-inverse and $\mathbf{k}_i = [k_{i1}, \ldots, k_{iN}]^T$, is the vector collecting the different numbers of phase rotations of each sources for the given sensor i. Each value of the vector $\mathbf{k}_i$ provides a different solution, so the result is written parametrized by $\mathbf{k}_i$

In the 2D-space, two sources are sufficient to obtain the different relative position vectors.
We propose here an empirical method, validated with test results in section 5, to find the unique solution (in most cases) in two steps:

- Step 1: considering a coarse model of array structure deformation, we can reduce the number of phase rotations from $k_{ij} \in \mathbb{Z}$ to an eligible one. For example in case of antenna mounted on wing structure we assume that the maximum deformation does not exceed few wavelengths: $k_{ij} \in \{-N_r, \ldots, N_r\}$, for all i, j. This model can be refined, for example, by considering that the array deformation can only be positive *i.e.* $k_{ij} \in \{-N_r, \ldots, N_r / \mathbf{p}_i'(\mathbf{k}_i) > 0\} \ldots$
- Step 2: consider C_1 =(source u_1, source v_1) a couple of sources ($u_1 \neq v_1$ are taken in {1,2...,N}). C_1 gives a family of eligible solutions that is to say the solutions of eq.(21) verifying the step 1. Let us denote $\{\mathbf{p}_i'(\mathbf{k}_i)\}_1$ this family. Another couple C_2 provides a different family of solutions $\{\mathbf{p}_i'(\mathbf{k}_i)\}_2$. The actual relative vector position $\mathbf{p}_i'$ belongs to each solution family. From N sources, we build $N-1$ different couples of sources. From a sufficient number of couples, the intersection of family of solutions conducts to an single element the actual position:

$$\mathbf{p}_i' = \bigcap_{l=1}^{N-1} \{\mathbf{p}_i'(\mathbf{k}_i)\}_l \tag{11}$$

In practice, in the 2D-space, 3 sources are almost always sufficient to identify the actual relative vector position (we met some pathologic cases where this empirical method fails).

4. Self-Calibration in Presence of Noise

Limitation of the Direct Method

Consider the initial continuous data model described in equation (1). The noise is assumed to be spatially white and equally powered on sensors and as a consequence the noise covariance matrix is scalar $\mathsf{E}[\boldsymbol{\eta}(t)\boldsymbol{\eta}^H(t)] = \eta^2\mathbf{I}_M$ where $\mathsf{E}[\,.\,]$ denotes the expectation operator.

In a first approach, $\mathbf{A}$ is supposed to be independent of time. The identification of constant array response matrix $\mathbf{A}$ is similar than in the free noise case except that we have to introduce second order statistics hoping to minimize the noise effects. The covariance matrix of the output signals is

$$\begin{aligned} \mathsf{E}[\mathbf{y}(t)\mathbf{y}^H(t)] &= \mathbf{A}\mathsf{E}[\mathbf{s}(t)\mathbf{s}^H(t)]\mathbf{A}^H + \eta^2\mathbf{I}_M \\ &= \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H \end{aligned} \tag{12}$$

where $\mathsf{E}[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the covariance matrix of the source signals, $\boldsymbol{\Lambda}$ and $\mathbf{U}$ are eigenvalue and eigenvector matrices of covariance matrice of the outputs.
As it is shown by Weiss et al. in [3] we have:

$$\mathbf{A}\mathsf{E}[\mathbf{s}(t)\mathbf{s}^H(t)]\mathbf{A}^H = \mathbf{U}_N\left[\boldsymbol{\Lambda} - \eta^2\mathbf{I}_N\right]\mathbf{U}_N^H, \tag{13}$$

where $\mathbf{U}_N$ is the $M \times N$ eigenvector matrix restricted to the N greatest eigenvalues of $\mathsf{E}[\mathbf{y}(t)\mathbf{y}^T(t)]$.
$\mathbf{A}$ can be decomposed again like equation (4):

$$\mathbf{A} = \mathbf{U}_N\mathbf{W}, \tag{14}$$

and its estimated is proceed using the Constant Modulus Approach described in the previous Section 3.

Now consider the case where $\mathbf{A}$ depends on time. $\mathbf{y}(t)$ is no longer stationary, and the ergodism property classically used to estimate covariance matrix from only one experiment has to be carefully used. We must distinguish two time scales.
First, when the duration of the experiment is short, the assumption of time independence for matrix $\mathbf{A}$ remains valid but the estimate covariance matrix of the noise is not a scalar matrix. Due to the consecutive estimation errors on the entries of $\mathbf{A}$, the research of phase rotation number in case of large deformation totally fails even for small errors.
Second, if the duration of the experiment is long, the covariance matrix of noise is accurately estimated, but $\mathbf{A}$ can no more be considered as a constant matrix.
Nevertheless, refining the data model with of an array deformation model, the non stationary problem can be split as two successive time-independent ones.

A Data Model adapted to Vibrating Antennas

For applications and observation durations we deal with, the deformation of the antenna is considered as the superposition of a large static bending (compared to the nominal shape) and low magnitude dynamic fluctuations due to vibrating modes as shown in fig.2. The position vector of sensor i is then parted into

$$\mathbf{p}_i(t) = \bar{\mathbf{p}}_i + \tilde{\mathbf{p}}_i(t),$$

where $\bar{\mathbf{p}}_i = [\bar{x}_i, \bar{z}_i]^T$, is the static position vector,
and $\tilde{\mathbf{p}}_i(t) = [\tilde{x}_i(t), \tilde{z}_i(t)]^T$ is the dynamic position vector. It is time-dependent and its temporal average is null.

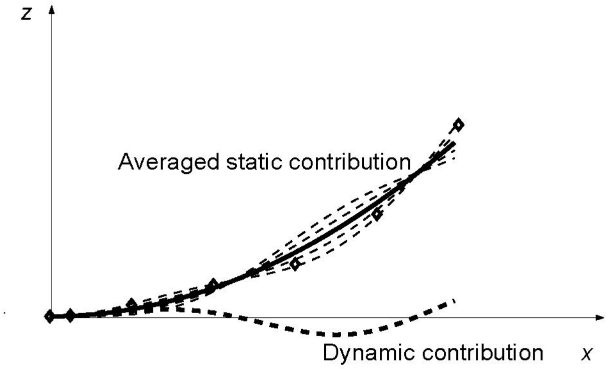

Figure 2. Array Shape Dynamic Model.

Replacing the position vector by its decomposition in expression (2), the entries of matrix $\mathbf{A}(t)$ become

$$\begin{aligned} a_{ij}(t) &= \exp\left\{ \mathrm{j}\frac{2\pi}{\lambda}\bar{\mathbf{p}}_i^T \mathbf{n}_j \right\} \exp\left\{ \mathrm{j}\frac{2\pi}{\lambda}\tilde{\mathbf{p}}_i^T(t)\mathbf{n}_j \right\}, \\ &= \bar{a}_{ij}(t)\tilde{a}_{ij}(t), \end{aligned}$$

or in matrix notation

$$\mathbf{A}(t) = \bar{\mathbf{A}} \circ \tilde{\mathbf{A}}(t), \tag{15}$$

where $\circ$ represents the Hadamard (element-wise) matrix multiplication.
Assuming that the dynamical distortions have low magnitude, the entries of dynamical array response matrix can be approximated as first order Taylor-series expansion:

$$\tilde{a}_{ij}(t) \simeq 1 + \underbrace{\mathrm{j}\frac{2\pi}{\lambda}\tilde{\mathbf{p}}_i^T(t)\mathbf{n}_j}_{f_{ij}(t)},$$

or in matrix notation

$$\tilde{\mathbf{A}}(t) \simeq \mathbb{1} + \mathbf{F}(t),$$

where

$\mathbb{1}$ is the $M \times N$ ones matrix such that $\mathbb{1}_{ij} = 1 \quad \forall i, j$
$\mathbf{F}(t)$ only account for first-order time dependent terms.

Hence

$$\mathbf{A}(t) = \bar{\mathbf{A}} + \mathbf{D}(t), \tag{16}$$

where $\mathbf{D}(t) = \bar{\mathbf{A}} \circ \mathbf{F}(\mathbf{t})$ is a deviation matrix depending of time.

Replacing $\mathbf{A}(t)$ by its expression (16), eq.(1) becomes

$$\mathbf{y}(t) = \left[\bar{\mathbf{A}} + \mathbf{D}(t)\right]\mathbf{s}(t) + \boldsymbol{\eta}(t). \tag{17}$$

which is the new data model adapted for vibrating antennas.

A Two Time Scales Method for Self-Calibration in presence of Noise

Introduction—The process is split in two steps.
The aim of the first step is to counteract the phase ambiguities by crosschecking several estimations provided by different subsets of source signals. This step being unfortunately white gaussian noise sensitive, a significant observation time is necessary (long time scale signal analysis). We only obtain a coarse shape close to the static one.
In the second step, the previous observation time is partitioned in small time intervals, each of them is used to obtain the almost instantaneous array shape from the estimated static one (short time scale signal analysis).
Note that the estimation of some static unknowns like the DOAs can be refined during the short time scale analysis.

First Step: estimation of the static array—Assuming that the signal vectors $\mathbf{s}(t)$ and the noise vector $\boldsymbol{\eta}(t)$, are realizations of stationary, zero means random process and there is no correlation between the noise and the signal, from equation (17) the records covariance matrix is

$$\begin{aligned} \boldsymbol{\Gamma}_{\mathbf{yy}}(t,t) &= \mathsf{E}\left[\mathbf{y}(t)\mathbf{y}^H(t)\right] \\ &= \left[\bar{\mathbf{A}} + \mathbf{D}(t)\right]\mathbf{R}_{\mathbf{ss}}\left[\bar{\mathbf{A}} + \mathbf{D}(t)\right]^H + \eta^2\mathbf{I}_M \\ &= \bar{\mathbf{A}}\mathbf{R}_{\mathbf{ss}}\bar{\mathbf{A}}^H + \bar{\mathbf{A}}\mathbf{R}_{\mathbf{ss}}\mathbf{D}^H(t) + \mathbf{D}(t)\mathbf{R}_{\mathbf{ss}}\bar{\mathbf{A}}^H \\ &\quad +\mathbf{D}(t)\mathbf{R}_{\mathbf{ss}}\mathbf{D}^H(t) + \eta^2\mathbf{I}_M, \end{aligned} \tag{18}$$

where$\mathbf{R}_{\mathbf{ss}}$ is the source signal covariance matrix and $\eta\mathbf{I}$ is the noise covariance matrix.
Applying the temporal averaging operator

$$\mathsf{m}\left[.\right] = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} . \quad dt$$

to the record covariance matrix, we can define a "stationarized covariance matrix" of the vector signal $\mathbf{y}$

$$\mathbf{R}_{\mathbf{yy}} = \mathsf{m}\left[\boldsymbol{\Gamma}_{\mathbf{yy}}(t,t)\right].$$

Because the dynamic deformation is temporally zero-mean, *i.e.* $\mathsf{m}\left[\mathbf{D}(t)\right] = 0$, averaging the equation (18) yields

$$\begin{aligned} \mathbf{R}_{\mathbf{yy}} &= \bar{\mathbf{A}}\mathbf{R}_{\mathbf{ss}}\bar{\mathbf{A}}^H + \mathsf{m}\left[\mathbf{D}(t)\mathbf{R}_{\mathbf{ss}}\mathbf{D}^H(t)\right] + \eta\mathbf{I} \\ &= \underbrace{\bar{\mathbf{A}}\mathbf{R}_{\mathbf{ss}}\bar{\mathbf{A}}^H}_{\text{order } 0} + \underbrace{\mathsf{m}\left[\bar{\mathbf{A}} \circ \mathbf{F}(t)\mathbf{R}_{\mathbf{ss}}\mathbf{F}^T(t) \circ \bar{\mathbf{A}}^H\right]}_{2^{nd}\text{ order matrix}} + \eta^2\mathbf{I}_M. \end{aligned} \tag{19}$$

Due to the temporal averaging, the order one terms of eq.(18) vanish in eq.(19). It remains only order 0 and order 2 terms in the power series development of $\mathbf{R}_{\mathbf{yy}}$. These 2 order terms will be neglected in the estimation of $\bar{\mathbf{A}}$ and the algorithms we use conduct to an estimated $\hat{\bar{\mathbf{A}}}$ such that

$$\mathbf{R}_{\mathbf{yy}} = \hat{\bar{\mathbf{A}}}\mathbf{R}_{\mathbf{ss}}\hat{\bar{\mathbf{A}}}^{\mathbf{T}} + \eta^2\mathbf{I}_{\mathbf{M}}.$$

similar to previous equation (12).
Once again the matrix $\hat{\bar{\mathbf{A}}}$ takes the form of equation (4)

$$\hat{\bar{\mathbf{A}}} = \mathbf{U}_{\mathbf{N}}\mathbf{W},$$

where $\mathbf{U_N}$ is now the matrix of eigenvectors corresponding to the N greatest eigenvalues of $\mathbf{R_{yy}}$.
The matrix $\hat{\bar{\mathbf{A}}}$ is estimated using CMA as described in Section 3 from eq.(4) to eq.(7).
Following, we compute the DOA θ_i, $i = 1, \ldots, N$ from equation (8).
We choose at least two distinct couple of sources from the N sources: $C_1 = (u_1, v_1) \in \{1, 2, \ldots, N\}^2$ $u_1 \neq v_1$, $C_2 = (u_2, v_2) \in \{1, 2, \ldots, N\}^2$ $u_2 \neq v_2$ and with $C_1 \neq C_2$. The estimation of eligible static relative position vector is then performed using equation (21):

$$\{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_1 = \frac{\lambda}{2\pi} \begin{bmatrix} \mathbf{n}_{u_1}^T \\ \mathbf{n}_{v_1}^T \end{bmatrix}^{-1} \begin{bmatrix} \bar{\phi}_{iu_1} + k_{u_1} 2\pi \\ \bar{\phi}_{iv_1} + k_{v_1} 2\pi \end{bmatrix}$$

$$\{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_2 = \frac{\lambda}{2\pi} \begin{bmatrix} \mathbf{n}_{u_2}^T \\ \mathbf{n}_{v_2}^T \end{bmatrix}^{-1} \begin{bmatrix} \bar{\phi}_{iu_2} + k_{u_1} 2\pi \\ \bar{\phi}_{iv_2} + k_{v_1} 2\pi \end{bmatrix}$$

where the $\bar{\phi}_{ij}$, $(j = u_1, v_1, u_2, v_2)$ are the phases extracted from the array reponse matrix $\hat{\bar{\mathbf{A}}}$.

The unique solution is then deducted by cross-checking the different families solutions $\{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_1$ and $\{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_2$ as in equation (11)

$$\bar{\mathbf{p}}_i = \{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_1 \cap \{\bar{\mathbf{p}}_i(\mathbf{k}_i)\}_2$$

Second step: Estimation of Dynamic Array Shape— In the previous calculus, we have neglected the second order terms in equation (19). Of course they are not negligible and, as a consequence, the static sensor locations we obtain are biased. However in this case, the relative deformation between the biased estimated static array and the actual one is small *i.e.* always lower than half a wavelength.
The very noise-sensitive stage relative to the research of phase rotation number in no more necessary. In this case we chose to estimate the covariance matrix of the outputs on a small duration experiment T_d considering the moving of the antenna is negligible. The temporal averaging is no more necessary and

$$\hat{\mathbf{\Gamma}}_{yy}(t) \simeq \hat{\mathbf{A}}(t)\mathbf{R_{ss}}\hat{\mathbf{A}}^H(t) + \eta^2\mathbf{I}_M,$$

where $\hat{\mathbf{\Gamma}}_{yy}(t)$ is the covariance matrix of the outputs estimated on the short time interval $[t, t + T_d]$ and $\hat{\mathbf{A}}(t)$ is the array response matrix associated to the array shape during the same time interval. Practically we will choose T_d less than 10% of the period of the most significant vibrating mode.
The matrix $\hat{\mathbf{A}}(t)$ is obtain performing a CMA on $\hat{\mathbf{\Gamma}}_{yy}(t)$ (see developments from eq.(4) to eq.(7)).

Now, the nominal known positions are the static position vectors $\bar{\mathbf{p}}_i$ estimated in the step 1, and the unknowns are the dynamic position vectors $\tilde{\mathbf{p}}_i(t)$.

$$\tilde{\mathbf{p}}_i(t) \triangleq \mathbf{p}_i(t) - \bar{\mathbf{p}}_i \quad i > 2. \tag{20}$$

We introduce $\hat{\phi}_{ij}(t)$ the phase deducted from the entry of $\hat{\mathbf{A}}(t)$

$$\hat{\phi}_{ij}(t) + k_{ij}2\pi = \frac{2\pi}{\lambda}\mathbf{p}_i^T(t)\mathbf{n}_j,$$

and $\bar{\phi}_{ij}(t)$ the phase deducted from the entry of $\bar{\mathbf{A}}(t)$ of step 1

$$\bar{\phi}_{ij} + k_{ij}2\pi = \frac{2\pi}{\lambda}\bar{\mathbf{p}}_i^T\mathbf{n}_j.$$

Because the number of phase rotations k_{ij} is the same in the two previous equations, the relative phase $\frac{2\pi}{\lambda}\tilde{\mathbf{p}}_i^T(t)\mathbf{n}_j$ is deduced without ambiguities from the measured phase $\hat{\phi}_{ij}(t)$ by

$$\frac{2\pi}{\lambda}\tilde{\mathbf{p}}_i^T(t)\mathbf{n}_j = \hat{\phi}_{ij}(t) - \bar{\phi}_{ij}, \quad i > 2,$$

hence

$$\mathbf{n}_j^T\tilde{\mathbf{p}}_i(t) = \frac{\lambda}{2\pi}(\hat{\phi}_{ij}(t) - \bar{\phi}_{ij}), \quad i > 2.$$

For a sensor i, the dynamic position vector is obtained from a number of sources greater or equal than the space dimension of the problem:

$$\tilde{\mathbf{p}}_i(t) = \frac{\lambda}{2\pi} \begin{bmatrix} \mathbf{n}_1^T \\ \vdots \\ \mathbf{n}_N^T \end{bmatrix}^\sharp \begin{bmatrix} \hat{\phi}_{i1}(t) + \bar{\mathbf{p}}_i^T\mathbf{n}_1 \\ \vdots \\ \hat{\phi}_{iN}(t) + \bar{\mathbf{p}}_i^T\mathbf{n}_N \end{bmatrix} \tag{21}$$

Finally, $\bar{\mathbf{p}}_i$ and $\tilde{\mathbf{p}}_i(t)$ being known, the current position vector $\mathbf{p}_i(t)$ is known for any sensor i and any time.

Note that during the evaluation of dynamic results, we can refine the static values of DOAs or static position vectors.

5. SIMULATION

Simulation 1

Background— We consider three sources at the same wavelength $\lambda = 30cm$ that are impinging on an irregular and incomplete array of omnidirectionnal sensors. The array is composed of $M = 8$ sensors. At rest, the coordinates x_i, z_i of the sensor i are given by

$$\begin{aligned} [x_i] &= [0\ 1\ 2\ 6\ 13\ 20\ 21\ 23]\frac{\lambda}{2}, \quad i = 1, \ldots, 8, \\ z_i &= 0, \quad i = 1, \ldots, 8. \end{aligned}$$

The DOAs of the $N = 3$ sources are $\theta_1 = -36°$, $\theta_2 = 3°$, $\theta_3 = 20°$. The noise sensor is injected with a Signal to Noise Ratio of 15 dB.
The number of snapshots used during the first step (static shape estimation), is $Ns = 6000$. We divide the observation time Ns in 60 blocks of 100 snapshots. The number of samples considers in the second step (dynamic shape estimation) is then $Nd = 100$.
The distortion follows a polynomial of degree 4. The magnitude of the distortion at the tip of the antenna is around 1.3λ .

So 3 sensors, 6, 7 and 8, have an ambiguous location.
Besides the static bending, we add a vibrating mode. The oscillation's frequency is $f_d = 10Hz$. The maximum magnitude is reached at the last sensor. The sampling frequency $fe = 60KHz$, allows us to observe the oscillation phenomenom during one period. The figure 3 details the array, with the positions of sensors at rest, and suffering from static plus dynamic distortions. The DOA's of the three sources are schematically represented too.

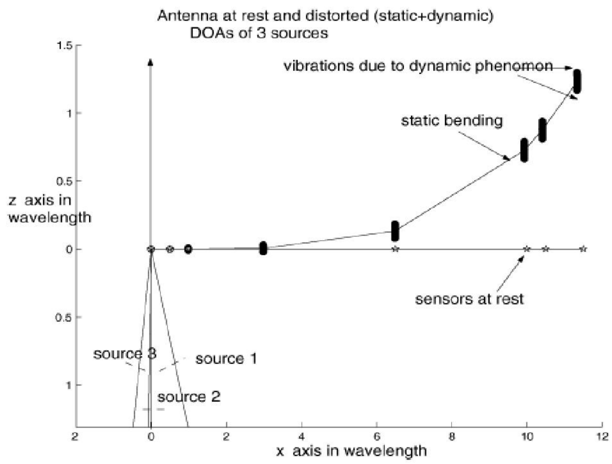

Figure 3. Array Shape Static and Dynamic Model.

Results—In the aim to expose the validity of the algorithm we have done 100 runs. The figure 4 represents the positions of the sensors obtained at the step 1. The marges of the vibration of the antenna are represented in solid line. We constat that the static estimation are biaised that confirm that the second order term is not negligeable in 19. We remember that,here, the goal is to eliminate the ambiguity of the three last sensors. We can check on this figure that for 100 runs, the array shape is approximatively estimated without ambiguity.

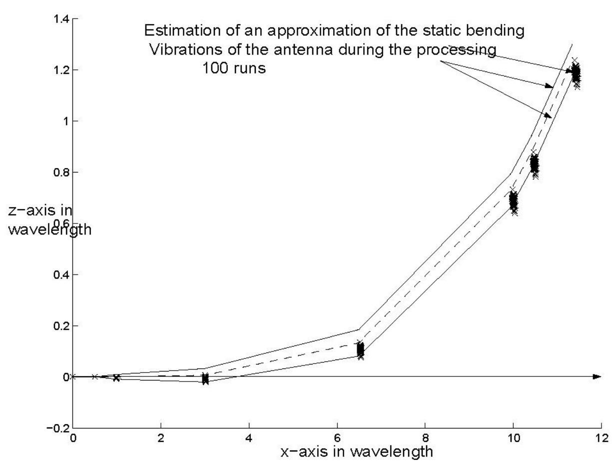

Figure 4. Positions obtained for 100 runs at the end of Step 1 .

The following Figure 5 illustrates the position of sensors 6,7 and 8 for the third samples block. The results are plotted together with the ellipses of confidence. These ellipses are computed from the Cramer Rao bound [2].

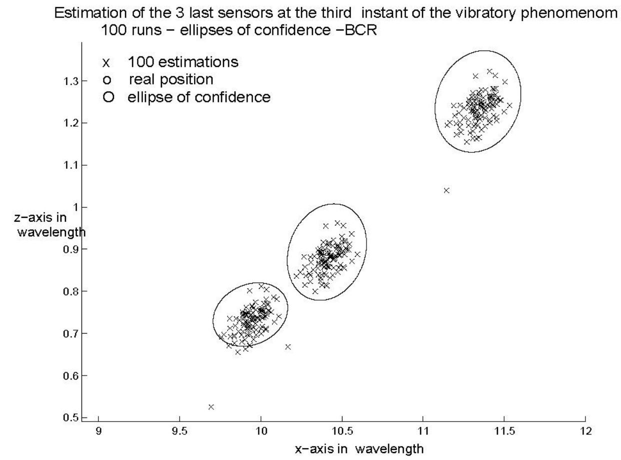

Figure 5. Ellipse of confidence and results

Second simulation

We present here a tracking result obtained in a less noisy context: $SNR = 30dB$. The DOAs and the carrier frequency λ are the same as previous simulation. The maximum static deformation is $4(\lambda/2)$. The dynamic one is about 10 percent $\lambda/2$. The antenna is composed of 7 sensors whose geometries at rest and statically deformed are illustrated in fig.6. The number of samples used for the static deformation estimation is N_S=10000 and the dynamic deformation is computed on 25 block of 400 samples. During this step, DOA estimations are refined taking into account the previous DOA estimations.

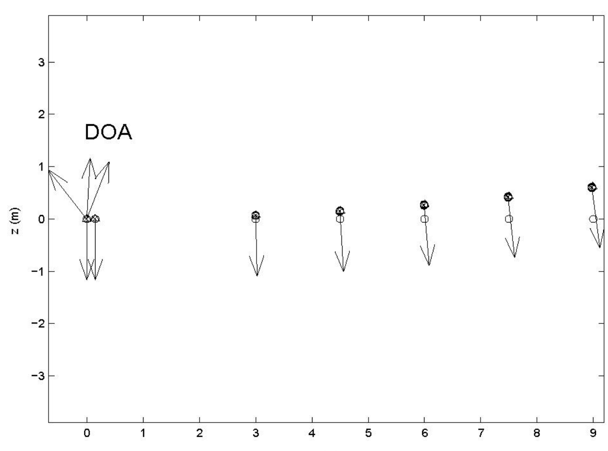

Figure 6. Antenna at Rest and Deformed Antenna.

The fig.7 is an illustration of tracking on the sensor 7 position. Starting from the estimated static position, we observe that the DOA refinement during the process improve the result.

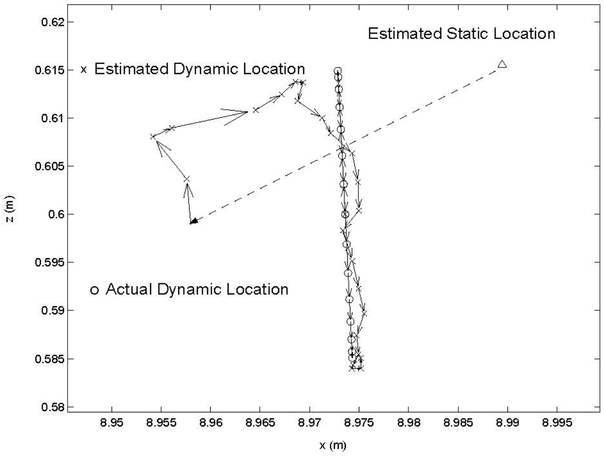

Figure 7. Tracking of Sensor 7 position

6. Conclusions

We presented a two-step method, based on Constant Modulus Algorithm, which succeeds in array shape self calibration. Ultimately, we have counteract the ambiguity level due to large bending. Furthermore, the tracking of the antenna during the vibrations has been achieved. To our knowledge, these two particular aspects has not already been land. Moreover, the first simulation results obtained confirm the validity of our approach.

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