Sensor Self-Calibration Methods For a Passive Conformal Airborne Antenna

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***Abstract*— This paper presents self-calibration methods applied to passive conformal airborne antenna. The sensors of the array are mounted under large wing of an aircraft. During the flight, sensors are subjected to important static distortions and dynamical vibrations so that the array must be self-calibrated in order to keep the optimal performance of localization. This self-calibration is globally a non-observable problem but it may be solved locally using a sufficient number of sources or mechanical distortion models. Two approaches of self-calibration methods are exposed. They allow solving for phase ambiguities and false positions problem due to deformations larger than half the wavelength.**

Airborne antenna array, localization of sources, sensor self-localization, passive systems

I. Introduction

During the last decade the number of antenna embedded by aerial vehicles has constantly increased [1]. With conformal antenna technology, the sensors are integrated on the surface of airplane. This installation does not impact the aerodynamic of the airplane and it allows taking advantage of the entire surface in order to increase the size of the antenna. For example, in the domain of airborne surveillance, and more particularly in direction finding of emitting sources at the ground level, large-sized sensor arrays are required in order to improve the antenna processing and increase the angular accuracy. For this kind of application, one solution is to benefit from the length of the wing span and integrate in it the array of sensors. In return, there is a major drawback. What we are mainly concerned with here is the flexibility of the wing and so the flexibility of the antenna. Indeed, during the flight, wings suffer static bending whose amplitude is linked, for example, to the quantity of petrol into tanks or the altitude of flight. At higher order of disturbance, the wing vibrates and ripples because of dynamical phenomena like flying control steering or environmental turbulences. Consequently, the antennas integrated into the wing are exposed to the same significant distortions. The exact locations of the sensors are consequently unknown.

But direction finding of emitting sources using a passive airborne sensor array requires the perfect knowledge of sensor locations. Self-calibration methods allow one to estimate sensor locations by using transmitters of opportunity at ground level.

Self-calibration methods are exposed in several papers assuming availability of multiple beacon signals, disjoint in time or in frequency [2], [3] or availability of a single but moving source [4]. The problem of sensor array self-localization where one considers multiple signals of opportunity, i.e. with unknown directions-of-arrival, observed at the same frequency cell and at the same time is studied in [5], [6], [7]. Rockah and Schultheiss, [2] showed that in a 2Dimensional problem, only three broadband sources are necessary for calibrating the array provided that the exact location of one sensor and the direction of another one are known, in the case of a small static bending. When non-disjoint sources are considered, Weiss and Friedlander presented a sensor self-localization method based on a maximum likelihood technique for narrowband sources and for omnidirectional sensors, [5]. They used a numerical routine that iterates alternatively between directions-of-arrival (DOA's) of sources and sensor locations until convergence. The initialization step is performed by the MUSIC algorithm, which is computed with sensor locations at rest.
In [5], the same authors developed a direct method called constant modulus algorithm (CMA), using the omnidirectionality characteristic of sensors in order to separately identify the element of the complex array response matrix in a direct process. In [7], Flanagan and Bell suggested a method for higher static distortion magnitude inspired from [5]. All the cited approaches assume static and small calibration errors i.e. smaller than the wavelength of the recorded 3 sources. When very high distortions are considered, ambiguity phase problems conduct these methods toward false locations close to locations at rest. Moreover, in case of vibrating arrays, iterative approaches [5], [7] can not converge.

This paper presents methods dealing with airborne conformal array shape self-calibration problem, based on persistent and non-cooperating sources such as TV, GSM or radio emissions. The paper is organized as follows: in Section II, the problem of an airborne antenna, the static and dynamic phenomena are presented. In Section III, The data model used to describe the recordings provided by a large distorted and vibrating array is exposed. In section IV, the local observability of the self-calibration problem is presented. In the last section, two methods developed in order to self calibrate a large sparse array are presented. A large static bending is assumed. The first method can be done with only three sources of opportunity and the use of a polynomial deformation or simpler physical constraints coupled with a method of phase ambiguities solving. Finally, to self-calibrate a large distorted and vibrating antenna an approach based on Constant Modulus Algorithm is suggested. Phase ambiguities are solved using a long integration time and then the track of the array shape during the vibration is possible by defining shorter integration times in a second step. Along the last section, numerical simulations and comparisons to Cramer Rao lower bounds are presented.

II. PROBLEM FORMULATION OF AN AIRBORNE CONFORMAL ANTENNA

In order to illustrate the paper, the kind of aircraft considered here is a motoglider. The sensors are inserted along the large and flexible aircraft wing. Its size can exceed 10 meters and the maximum deformation can reach up to 60 cm at the wing tip [10], i.e., greater than the typical wavelengths (λ). Conversely, we assume that deformations near the fuselage are negligible. Generally, only the very low frequency vibrating modes (<10 Hz) are significant. Their magnitudes can roughly reach up to 10% of the maximum static deformation in steady flight, i.e., lower than the typical wavelengths. Fig. 1 illustrates these two phenomena of distortion.

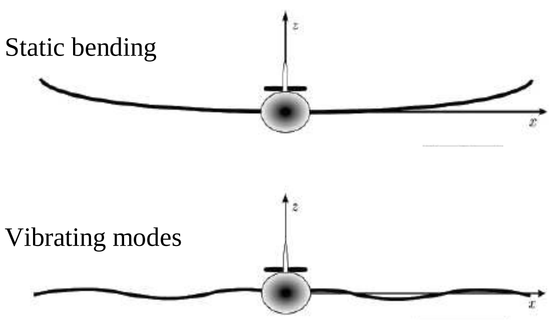

Figure 1. Illustration of static and dynamic phenomena

III. DATA MODEL

The axis system is described in Fig. 2.

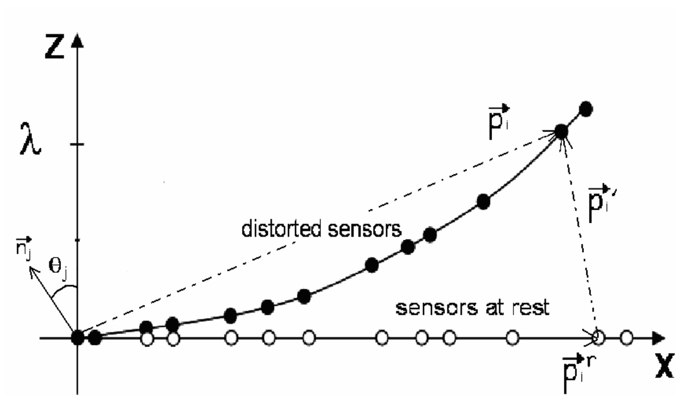

Figure 2. Axes system

A. Sources

We consider N narrowband sources with the same carrier frequency (wavelength λ). The number of sources is assumed to be known. The DOA of sources is given by the matrix **N**

$$\mathbf{N} = [\mathbf{n}_1^T \dots \mathbf{n}_N^T]^T, \quad (1)$$

each column of **N** is a unit vector $\mathbf{n}_j$ associated to a source

$$\mathbf{n}_j = [\sin\theta_j, \cos\theta_j]^T. \quad (2)$$

All unit source vectors are assumed distinct. The different sources are assumed to be ergodic, stationary and zero mean random processes. All sources are land-based and they are in the same half-plane, $-\pi/2 < \theta_j < \pi/2$, for all $j \in \{1, \dots, N\}$. The DOA of each source is the same for all sensors (far-field assumption). We assume there is no multi-path as the signals originate from the ground.

B. Sensors arrays

The array is composed of M omnidirectional and identical sensors ($M > N$). The position of the first two sensors is known and is used to define the origin and the x-axis of the coordinate system, see Fig. 2. The distance between them is lower than or equal to half the wavelength λ of the carrier frequency. We assume that during the process the position of a sensor is time-dependant. In this coordinate system, the current position vector of the i^{th} sensor is defined by

$$\mathbf{p}_i[n] = [x_i[n], z_i[n]]^T \quad (3)$$

Nominal sensor locations (for example corresponding to the antenna at rest) are known. The inter-sensor distance, except the distance between the first two sensors, can be greater than λ/2. In the same way, the distance between the current location and at rest location of a sensor can be greater than λ/2; this justifies the expression "large deformations".

C. Array response matrix

The matrix **A**[n] stands for the (M×N) array response matrix. The columns of **A**[n] are the steering vectors of each source.

A[n] is full column rank. Since the sensors are omnidirectional and identical, the modulus of each component of the matrix **A**[n] is constant. It is taken equal to one without loss of generality. Sources and sensors are assumed in the same geometrical plane.
Under these assumptions, an entry $a_{ij}[n]$ of **A**[n] can be expressed as

$$a_{ij}[n] = \exp(j\phi_{ij}[n]) \tag{4}$$

with

$$\phi_{ij}[n] = -\frac{2\pi}{\lambda}\mathbf{p}_i^T[n]\,\mathbf{n}_j\,. \tag{5}$$

The first sensor is the origin of the system. So the first line of the array response matrix is equal to one.

D. Recordings

The signal at the output of the sensors can be described by the M-dimensional vector $\mathbf{y}[n] = [y_1[n], \dots, y_M[n]]^T$ as

$$\mathbf{y}[n] = \mathbf{A}[n]\mathbf{s}[n] + \boldsymbol{\eta}[n] \tag{6}$$

where $\mathbf{s}[n] = [s_1[n], \dots, s_N[n]]^T$ is the vector of the N baseband sources and $\boldsymbol{\eta}[n]$ is the noise vector. By assumption, each component $\eta_i[n]$, (i = 1,…, M) of the noise is an ergodic, stationary and zero mean random process. Moreover, it is spatially and temporally white with the power η^2 on each sensor. To facilitate readers understanding, the reference to the sample (n) will be suppressed.

IV. Local Observability of Self-Calibration

In this part, the observability conditions of self-calibration are exposed. Many assumptions are made. First the array response matrix is supposed to be estimated [5-9]. The assumption of no ambiguity of the direction of arrival is done. We show that there is not a unique position candidate for each sensor. Few positions give the same records. Consequently, we demonstrate that is it possible to find a unique solution only for local observability assumptions which will be exposed.

A. Observability of self-calibration of small and large distorted antenna

As it is introduced in [2], even if the antenna is small or no distorted and even if the phase in (5) is exactly computed, the knowledge of the position of two sensors is a necessary condition, and that, for three reasons.

First, due to the far-field assumption, a vector $\mathbf{n}_j$ is independent of any translation in space. So, the position of one sensor must be known.

Secondly, a rotation applied on the entire axis system (i.e. the same rotation for the position and the DOA) would have no effect on the computation of phases. In fact, considering **U** a rotational matrix

$$\begin{aligned}\phi_{ij} &= -\frac{2\pi}{\lambda}\mathbf{p}_i^T\mathbf{U}^T\mathbf{U}\mathbf{n}_j \\ &= -\frac{2\pi}{\lambda}\mathbf{p}_i^T\mathbf{n}_j\end{aligned} \tag{7}$$

So, the knowledge of the orientation of a second sensor is required.

Thirdly, in the particular case of a linear antenna, a rotation on the DOA can be compensated by a homothety on the antenna array, as it is illustrated on Fig. 3. So the distance between two sensors must be known.

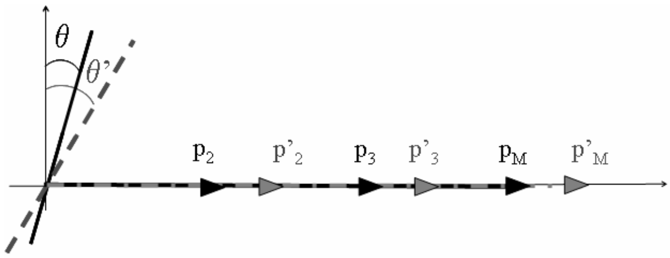

Figure 3. Homothety phenomena

In order to self-calibrate an antenna, the necessary condition is to know the position of one sensor, the orientation of a second one and the distance between two sensors. Indeed, we consider in this paper that the positions of the first two sensors are known.

B. Local observability of self-calibration of a large distorted antenna

Each entry of **A** depends on two parameters, the current positions of the sensor and the DOA of the source (4), (5). The assumption of previously estimated DOA is done. It is an exponential function of the phase, so it is only possible to extract the argument given in [-π,π]. A rotation of the phase will be obvious (8), that get the correct position $\mathbf{p}_{i\_cor}$ and ambiguous positions $\mathbf{p}_{i\_amb}$ confused (9).

$$\phi_{i\_cor} = \phi_{i\_amb} + \mathbf{k}_i 2\pi \tag{8}$$

$$\frac{2\pi}{\lambda}\mathbf{p}_{i\_cor}^T\mathbf{N} = \frac{2\pi}{\lambda}\mathbf{p}_{i\_amb}^T\mathbf{N} + \mathbf{k}_i 2\pi \tag{9}$$

In case of large distortion, the self-calibration problem is not observable. By increasing the number of sources or by using to take into account additional physical considerations, it is possible to reduce the number of ambiguous positions or render it locally observable.

V. METHODS OF SELF-CALIBRATION

Two methods of self-calibration will be exposed. The first one is used as an initialization of a classical self-calibration processing, like e.g., [5]. It uses a polynomial model of distortion. The second method is an adaptation of the self-calibration method of [6]. An additional step of phase ambiguities solving is proposed. It uses physical approaches of distortion when the environment is noisy. These two methods allow dealing with antenna suffering large distortion.
Consequently, contains for the two methods will be:

A. First method : MUSIC-M

Assumption 1: *The distortion of the antenna follows a polynomial model.*

This method is used to initialize the self-calibration algorithm. It is based on the classical high resolution source localization algorithm, MUSIC [11].

For each sensor and for many values of the coefficient α, couples of coordinates are computed using the following polynomial model of distortion

$$\begin{cases} z_i = \alpha\, {x_i^r}^4, & (10) \\ \sqrt{(z_i - z_{i-1})^2 + (x_i - x_{i-1})^2} = x_i^R - x_{i-1}^R & (11) \end{cases}$$

For each value of coordinates, the steering vector is computed, that allows plotting the MUSIC-M pseudo spectrum in two-dimension, Fig.4.

$$f_{music-M}(\theta,\alpha) = \frac{a(\theta,\alpha)^H a(\theta,\alpha)}{a(\theta,\alpha)^H E_b E_b^H a(\theta,\alpha)} \quad (12)$$

Fig. 4 corresponds to the MUSIC-M pseudo-spectrum for 3 sources of DOA -36°, 3°, 20°. The last sensor has a distortion of 2.67 λ/2.

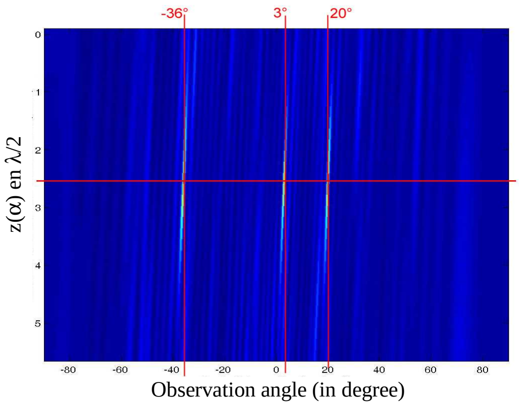

Figure 4. MUSIC-M pseudospectrum

B. Second method : Phase ambiguities solving

The current position vector is equal to the sum of the known position at rest and the unknown difference between the two due to distortion, see Fig.2. In case of large bending, the difference of position can be expressed by

$$\mathbf{p}_i'(\mathbf{k}_i) = \frac{\lambda}{2\pi}\, \mathbf{N}^{\#}\, [\arg(\mathbf{a}_i./\, \mathbf{a}_i^r) + \mathbf{k}_i 2\pi], \quad (13)$$

with $\mathbf{a}_i^r$ the steering vector of sensor i computed with the position at rest, $\mathbf{a}_i$ the steering vector of the distorted sensor, and $\mathbf{k}_i$ the unknown vector of the numbers of phase rotations for all sources between its current position $\mathbf{p}_i$ and its position at rest $\mathbf{p}_i^r$.

1) Without noise

Assumption 2: *Due to the structure of the antenna, the number of phase rotations* $\mathbf{k}_i$ *lies in a finite subset* $D=\{-N_k, \dots, N_k\}^N$, *with* $N_k \in \mathbb{N}$.

The aim is to determine $\mathbf{k}_i$ by applying the following algorithm:

1. For each $\mathbf{k}_i \in D$, compute $\mathbf{p}_i'(\mathbf{k}_i)$ according to (13)
2. Verify if $\mathbf{N}\mathbf{p}_i'(\mathbf{k}_i) - \frac{\lambda}{2\pi}[\arg(\mathbf{a}_i./\mathbf{a}_i^r) + \mathbf{k}_i 2\pi] = 0$

If there is equality, $\mathbf{k}_i$ is the correct number of phase rotation and $\mathbf{p}_i = \mathbf{p}_i^r + \mathbf{p}_i'(\mathbf{k}_i)$
Otherwise, go to step 1.

Algorithm 1

2) With Noise

In addition to the assumption 2, we consider that the relative intersensor distance deformations are small and lower than a threshold ε (since we are in the case of an airborne antenna), i.e.,

$$\frac{\left\| \mathbf{p}_{i+1} - \mathbf{p}_i \right\| - \left\| \mathbf{p}^r_{i+1} - \mathbf{p}^r_i \right\|}{\left\| \mathbf{p}^r_{i+1} - \mathbf{p}^r_i \right\|} \le \varepsilon \quad (14)$$

We consider that a too large distortion can not be realist. We verify if the angle formed by three sensors is not too important, Fig 5.

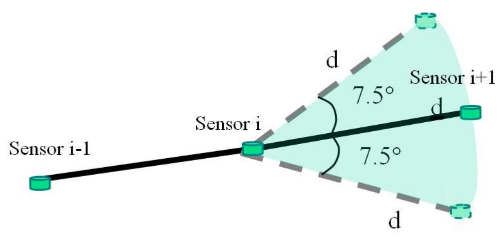

Figure 5. Physical constraints

Indeed, an additional step is added in algorithm 1 to verify if the physical constraints are respected. In this way, the infeasible calculated positions can be discarded.

C. *Simulations results*

To test these two methods, simulations have been performed. They have been based on the following example of antenna. It is an array composed of 13 sensors. At rest, the coordinates of the sensors are in half-wavelength.

TABLE I. COORDINATES OF THE SENSORS

$\mathbf{x}^r$	0	1	5	8	12	21	30	39	48	53	54	56	58
$\mathbf{z}^r$	0	0	0	0	0	0	0	0	0	0	0	0	0

The distorted antenna is represented by the solid black line in Figure 6 and Figure 7.

1) *Self-calibration results with MUSIC-M*

Figure 6 represents 500 results of an iterative classical self-calibration method like [5] after an initialization with MUSIC-M algorithm.

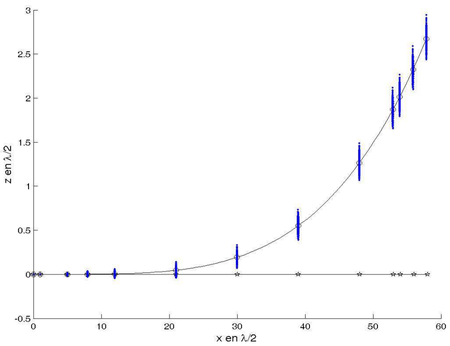

Figure 6. 500 Self-calibration results after MUSIC-M

2) *Self-calibration results with the phases ambiguities solving.*

Figure 7 represents 500 results of a direct sub-space/ constant modulus method, like [6],[8] and [9]. We add algorithm 1 associated to physical constraint in order to solve the phase ambiguities.

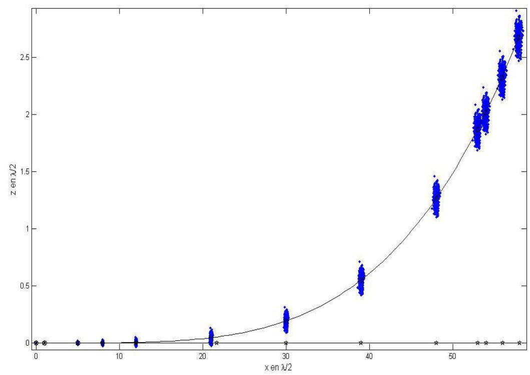

Figure 7. 500 Self-calibration results with a direct method algorithm 1

CONCLUSION

This paper presents the problem of large distorted conformal airborne antenna. The observability of self-calibration problem is exposed. Two methods have been developed to estimate positions of sensors suffering from large distortion. Results of simulations will be presented. The antenna is correctly self-calibrated. All the ambiguities positions have been eliminated. The first iterative method is more computation-intensive and needs a model of distortion, even if it could be very simple. The second method is direct and requires only physical constraints in the case of a noisy environment.

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