Improving BEM channel estimation for airborne passive radar reference signal reconstruction

Clément Berthillot[*,**], **Agnès Santori**[*], **Olivier Rabaste**[***], **Dominique Poullin**[***], **Marc Lesturgie**[**,***]

[*]French Air Force Research Center, 13661 Salon Air, France
email: clement.berthillot@intradef.gouv.fr

[**]SONDRA, Gif-sur-Yvette Cedex, France

[***]ONERA, The French Aerospace Lab
BP 80100, 91123 Palaiseau Cedex, France

Abstract: *In this article, we consider an airborne passive radar using ODFM signals of opportunity in a Single Frequency Network (SFN) environment. The long and sparse multipath fading channel and the mobility of the receiver can deeply alterate the signal and therefore degrade the decoding processing. Classic methods experience difficulties in dealing with such channel impacts. Consequently we present a Basis Expansion Model (BEM) based channel estimation method. We also propose to combine it with the Minimum Description Length (MDL) and ESPRIT algorithms to get a prior information. Simulation results emphasize the benefits of using the BEM and confirm the necessity to have an a priori knowledge of the channel.*

1. Introduction

Recently research and studies have led to the development of efficient and reliable ground passive radars using OFDM signals. As with most radars, passive radar processing is based on the well known matched filter that requires the transmitted signal knowledge. Thus the first mandatory step of passive radar systems is to estimate this reference signal. For passive ground radar where the propagation channel multipaths are not subject to Doppler shifts, this estimation takes advantage of the particularities of the OFDM signals to decode the transmitted information.

Unfortunately, the aeronautical environment introduces new perturbations which make ineffective the classic interpolation-based methods. The mobility of the receiver implies indeed time variations of the channel, that may induce a large Doppler spread, and as a consequence troublesome InterCarrier Interference (ICI). Besides, the Single Frequency Network (SFN) accentuates the composite nature of the received signal, which may contain equivalent contributions of several broadcasters. In case of poor estimation, the reference signal remains composite, which leads to duplication of detected targets.

After pointing out the disturbing impact of the channel, we present a Basis Expansion Model (BEM) - based technique to estimate the channel. We then propose a preliminary channel parameter estimation step to adapt the method to the aeronautical environment. Finally we comment on simulation results of our estimation algorithm.

2. Airborne Passive Radar Constraints

According to OFDM principles, first binary data are mapped into elementary symbols of a complex constellation. N elementary symbols constitute an OFDM symbol $\mathbf{X}$. Among these N subcarriers are inserted L pilots that are known at the receiver. Let P_m and Υ_m for $m = 0 \dots L-1$, denote respectively their index and value. The time domain equivalent symbol is inferred from $\mathbf{X}$ by applying an inverse discrete Fourier transform: $\mathbf{x} = \mathbf{F}^H \mathbf{X}$, where $\mathbf{F}$ is the unitary discrete Fourier matrix, and $.^H$symbolizes the hermitian transpose. A guard interval, composed of the last G samples of the symbol, is concatenated to the front of $\mathbf{x}$. Let T_s denote the sample period. The signal is then sent over the channel.

The aeronautical channel can be modeled using several elements: first a Line-Of-Sight (LOS) component can be considered since this direct path is seldom masked in airborne passive radar. As passive radar usually profits from terrestrial broadcasters, such as TV or radio, that are tilted toward the ground, we also consider ground reflections (GR). Besides, reflections on the ground obstacles may introduce diffuse backscattered components. So the channel is composed of multiple paths, and the received signal is consequently strongly composite. As a consequence, the Channel Impulse Response (CIR) at time instant iT_s is modeled as: $h(iT_s, \tau) = \sum_{r=1}^{\nu} h_{(i,\tau_r)}^{(t)} \delta(\tau - \tau_r)$, where $h_{(i,\tau_r)}^{(t)}$ are the complex path gains, τ_r the path delays, and ν the channel length. The channel filters the signal, so that the received one can be written in a matrix way: $\mathbf{y} = \mathbf{H}^{(t)}\mathbf{x} + \mathbf{u}$, where $\mathbf{u}$ is an additive white Gaussian noise, and where one diagonal of $\mathbf{H}^{(t)}$ corresponds to one delay, that is to say, to one channel tap along time. In the frequency domain:

$$\mathbf{Y} = \mathbf{F}\mathbf{y} = \mathbf{H}\mathbf{X} + \mathbf{U} \tag{1}$$

where $\mathbf{H} = \mathbf{F}\mathbf{H}^{(t)}\mathbf{F}^H$ is the channel matrix, and $\mathbf{U}$ remains white Gaussian.

But the aeronautical channel is also characterised by the mobility of the receiver. It induces a Doppler shift, different for each path, and as a consequence InterCarrier Interference (ICI). In case of a fixed ground receiver, the channel would be time invariant, and so $\mathbf{H}^{(t)}$ a circulant matrix. As a consequence, the channel matrix $\mathbf{H}$ could be assumed to be diagonal. That is why classic methods aim at estimating only this vector at pilot indices: $\hat{\mathbf{H}}(P_m, P_m) = \mathbf{Y}(P_m)/\Upsilon_m$, for $m = 0 \dots L-1$. But in the present case, $\mathbf{H}^{(t)}$ is no longer a Toeplitz matrix. The off-diagonal terms represent the way the subcarriers interfere with one another. So the channel matrix can be seen as the subcarrier coupling matrix. According to (1), the observed sample on the k^{th} frequency $\mathbf{Y}(k)$ is a mix of the neighbooring elementary symbols of $\mathbf{X}(k)$:

$$\mathbf{Y}(k) = \mathbf{H}(k,k)\mathbf{X}(k) + \sum_{\substack{i=0 \\ i \neq k}}^{N-1} \underbrace{\mathbf{H}(k,i)\mathbf{X}(i)}_{\text{Interference from } \mathbf{X}(i) \text{ on } \mathbf{Y}(k)} + \mathbf{U}(k) \tag{2}$$

In fact, the Doppler shift has to be considered regarding the OFDM intercarrier bandwidth $\Delta_f = 1/(NT_s)$. Thus ICI effect is all the more important that the ratio $f_{dT} = f_D/\Delta_f$, called normalized Doppler, is large.

3. BEM Estimation Model

The BEM estimation principle consists in decomposing the CIR on a basis containing $Q+1$ functions to model the taps time variations [1], [2]. The r^{th} channel taps $h^{(t)}_{(.,r)}$ over one OFDM symbol is described by the BEM coefficients $h^{(b)}_{(.,r)}$:

$$\begin{bmatrix} h^{(t)}_{(0,r)} \\ \vdots \\ h^{(t)}_{(N-1,r)} \end{bmatrix} = \mathbf{B} \begin{bmatrix} h^{(b)}_{(0,r)} \\ \vdots \\ h^{(b)}_{(Q,r)} \end{bmatrix} + \epsilon \tag{3}$$

with $\mathbf{B}$ the $N \times (Q+1)$ basis matrix, and ϵ the modeling error. We introduce $\mathbf{h}_q = [h^{(b)}_{(q,0)}, \ldots, h^{(b)}_{(q,\nu-1)}]^T$ the decomposition coefficients for the q^{th} function, $q = 0 \ldots Q$, and $\mathbf{h} = [\mathbf{h}_0^T, \ldots, \mathbf{h}_Q^T]^T$ that collects all the model coefficients to estimate, where $.^T$ is the transpose operation. So the channel estimation is turned into the decomposition coefficient estimation. The BEM benefit consists in evaluating only $(Q+1) \times \nu$ parameters instead of the $N \times \nu$ taps, with $Q \ll N$. Several basis $\mathbf{B}$ exist. We use here the Polynomial BEM (P-BEM) which decomposes the channel taps as a linear combination of polynomials of increasing degree from 0 to Q. The entry of $\mathbf{B}$ corresponding to row p and column q is defined by: $\mathbf{B}(p,q) = (p+1)^q$

The BEM model includes the observed samples at the pilot index, but also its neighbooring samples. For each pilot at index P_m, we consider: $\mathbf{Y}_m = [\mathbf{Y}(P_m - \gamma), \ldots, \mathbf{Y}(P_m + \gamma)]^T$, where the γ parameter allows to adapt the estimation model to the amount of ICI it has to deal with. All the observation vectors $\mathbf{Y}_m$ are gathered together in one vector $\mathbf{Y}^{(p)}$ and expressed versus $\mathbf{h}$. As detailed in [1]:

$$\mathbf{Y}^{(p)} = \mathbf{D}^{(p)}\mathbf{S}^{(p)}\mathbf{h} + \mathbf{d} + \mathbf{v} \tag{4}$$

where $\mathbf{D}^{(p)}$ consists of the extractions of $\mathbf{B}$ in the frequency domain corresponding to all $\mathbf{Y}_m$ indices, and $\mathbf{S}^{(p)}$ contains the pilot values and the channel knowledge, especially its length. In the following, $\mathbf{D}^{(p)}\mathbf{S}^{(p)}$ is denoted $\mathbf{P}$. $\mathbf{d}$ depends on the remaining data and $\mathbf{v}$ expresses at once the channel additive noise and the modeling error.

As the aeronautical channel varies rapidly, the BEM estimator needs to contain enough functions to describe its fluctuations. Besides, we suppose the channel can be long. So the number of coefficients in the BEM basis remains high although it is far lower than the whole channel matrix non-zero terms, all the more so it is bounded by the observation number: $(Q+1)\nu \leq (2\gamma+1)L \leq N$.

Finally, from (4), [1] derives the Linear Minimum Mean Square Error (LMMSE) estimator:

$$\hat{\mathbf{h}}_{LMMSE} = \mathbf{R}_h \mathbf{P}^H \left(\mathbf{P}\mathbf{R}_h\mathbf{P}^H + \mathbf{R}_d + \mathbf{R}_v\right)^{-1} \mathbf{Y}^{(p)} \tag{5}$$

where $\mathbf{R}_h$, $\mathbf{R}_d$, $\mathbf{R}_v$ stand respectively for the correlation matrix of the channel taps, the data and the noise. These matrices need to be evaluated in accordance to statistical assumptions. Especially the channel taps are commonly modeled as a Wide-Sense-Stationary Gaussian process. Each path are supposed independent from one another, the scatterers are said uncorrelated. And finally, the time correlation for one tap is assumed to respect Jakes' model [4].

4. Method improvement by prior channel parameter estimation

Considering the sparsity of the channel, only a few paths are active among the $(Q+1)\nu$ estimated ones. We noticed that the inactive taps are non zero estimated. So, their even poor but numerous contributions deeply corrupt the demodulation process. Therefore we propose to seek for the only active taps.

As proposed in [5], we first determine the path number using the Minimum Description Length criteria. Derived from the information theory, it aims at representing the signal information with the most relevant number of sinusoidal signals. The algorithm is based on the observation of the channel matrix at pilot index P_m: $\mathbf{H}^{obs} = [\mathbf{Y}(P_0)/\Upsilon_0, \ldots, \mathbf{Y}(P_{L-1})/\Upsilon_{L-1}]$. $\hat{\nu}$ corresponds to the multiplicity of the smallest eigenvalue of the correlation matrix of the channel observation, [6], [7]. It can be noticed that, for each pilot at index P_m:

$$\mathbf{Y}(P_m)/\Upsilon_m = \sum_{r=1}^{\hat{\nu}} \alpha_r \left(e^{-j2\pi\frac{\tau_r}{LTs}}\right)^m + \mathbf{U}(P_m)/\Upsilon_m \tag{6}$$

So the $\hat{\nu}$ delays can be evaluated by applying ESPRIT algorithm to $\mathbf{H}^{obs}$, where the estimated poles $z_r = e^{-j2\pi\frac{\tau_r}{LTs}}$ for $r = 1 \ldots \hat{\nu}$ give access to the delays:

$$\hat{\tau_r} = (LT_s \arg(z_r)) \,/\, (2\pi) \tag{7}$$

This prior channel knowledge, $\hat{\nu}$ and $\hat{\tau_r}$, can directly be included into the BEM design of $\mathbf{S}^{(p)}$ from (4).

Nevertheless (7) points out a restriction of the method. Indeed, $\arg(z_r)$ lies between 0 and 2π. Therefore the maximum delay that can be measured is LT_s. It corresponds to suppose that the channel is not longer than L samples. Generally, the channel is supposed smaller than the guard interval G, so as to avoid intersymbol interference. Depending on G, our method may be more restrictive.

5. Simulation Results

The simulated signal and channel parameters used to illustrate our method are summarized in Tab.1 and Tab.2. Note that in the scope of this article, we simulate delays that are multiple of the sample period. According to [1] and to the boundaries detailled in section 3, we choose $Q = 4$, and $\gamma = 6$.

DVB-T mode 2K	N	2048
	L	176
	G	256
Constellation		16-QAM
SNR		30 dB

Table 1 – Simulated signal

Transmitter 1	LOS	$0 \times T_s$	$-2.7dB$
	GR	$1 \times T_s$	$-6.5dB$
Transmitter 2	LOS	$140 \times T_s$	$-7.7dB$
	GR	$141 \times T_s$	$-11.5dB$
Diffuse component		$100 \times T_s$	$-26.3dB$

Table 2 – Simulated channel delay and power

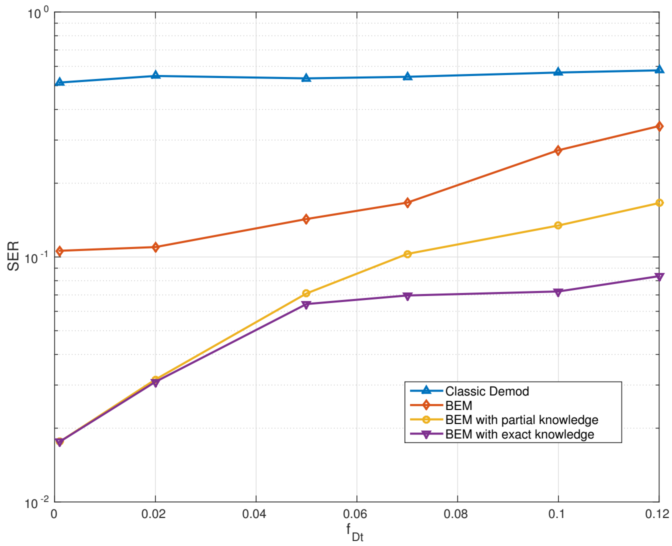

Figure 1 – Symbol Error Rate

At first we run the classic interpolation as evoked in section 2. We compare it to the BEM algorithms as described in section 3, and section 4. Concerning the BEM proposed in section 4, we considered two different implementations. The first one estimates the path number with MDL. Since this estimate proved to be very reliable on simulations considering delays on the sample grid and small Doppler shifts, we denoted this method by "BEM with exact knowledge". However it may be less efficient with real delays or larger Doppler shifts [5], although this cannot be demonstrated here.So we also considered an intermediary method, denoted "BEM with partial knowledge", that represents the possibility that the MDL method provides an erroneous estimate of the path number when facing a real and more complex channel. Here we set $\nu = 20$, in accordance to the channel model of section 2, and we assume in this case that the ESPRIT method estimates delays for the active paths as well as some inactive ones. Indeed this method aims at reducing the number of paths to estimate, even if the exact number is not known.

To measure the performance of the BEM estimations we compute the Symbol Error Rate (SER) after LMMSE equalization versus f_{dT}, as introduced in section 2. Fig.1 highlights the inability of the classic method to capture the channel composite nature, contrary to the BEM. Moreover the channel prior knowledge leads to a significant channel estimation improvement and as a consequence, a better reference signal estimation, even if the channel parameters are only partially known. Without prior channel information, the inactive path misestimation degrades the method performance. Fig.2 shows the estimated constellations obtained with the four methods.

6. Conclusion

The concept of airborne passive radar is a very recent topic of research. Current methods to estimate the direct path do not satisfy the constraints imposed by the receiver platform mobility, especially ICI. On the contrary, BEM methods are designed to take it into account. Used on its

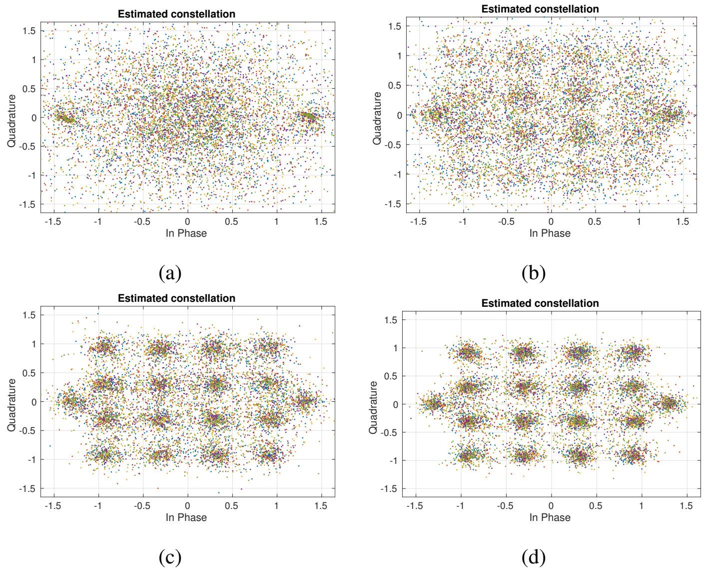

Figure 2 – Estimated constellation for $f_{dT} = 0.1$: Classic interpolation (a) , BEM method (b), BEM with partial (c) and exact (d) path knowledge.

own, simulation results showed a better efficiency, but still limited, than classic interpolation. However, the MDL-ESPRIT initialization step provides sufficient knowledge to overpass the channel sparsity constraint, and proved significant improvements, even if there could be limits like the channel length. Experimental trials will validate the assumed channel model and the relevance of the methods detailed in the scope of this article.

References

[1] Z. Tang, RC. Cannizzaro, G. Leus and P. Banelli, “Pilot-Assisted Time-Varying Channel Estimation for OFDM Systems”, IEEE Transactions on Signal Processing, 2007, vol. 55, no. 5, pp. 2226-2238.

[2] M. Poggioni, L. Rugini, and P. Banelli, “Multistage decoding-aided channel estimation and equalization for DVB-H in single-frequency networks”, Signal Processing Advances in Wireless Communications, June 2009, IEEE 10th Workshop on, pp. 181-185.

[3] M. Visintin, “Karhunen-Loeve expansion of a fast Rayleigh fading process”, IEEE Electronic Letters, August 1996, vol. 32, no. 8, pp 1712-1713.

[4] J. Proakis and M. Salehi,“Digital Communications”, 5th Edition, 2007, ed. McGraw-Hill, chap. 13, pp 838-839

[5] B. Yang, K. Letaief, K. Ben and R. Cheng, and Z. Cao, “Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling”, IEEE Transactions on Communications, 2001, no. 3, pp. 467-479.

[6] M. Wax and T. Kailath, “Detection of signals by information theoretic criteria”, Acoustics, Speech and Signal Processing, 1985, vol. 33, no. 2, pp. 387–392.

[7] G. Xu, R. H. Roy III, T. Kailath, “Detection of number of sources via exploitation of centro-symmetry property”, IEEE Transactions on Signal Processing, 1994, vol. 42, no. 1, pp. 102-112.