Wavelet versus Fourier for wireless SAW sensors resonance frequency measurement

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***Abstract*—The purpose of this communication is to show that wavelet transform provides a relevant way to the Fourier transform in the case of central frequency measurement of a SAW resonator. After recalling the limits of the Fourier transform, we present a method based on wavelet transform modulus maxima which allows staying in the time domain without using the frequency domain. We show that this approach is a way to improve the spectral resolution and we propose a technique, checked on experimental signals, able to measure the SAW resonator frequency according to the desired spectral resolution.**

I. Motivation and objective

SAW resonators are more and more used as wireless and batteryless sensors [1]. Both time domain and frequency domain readers are in competition to determine the resonance frequency of the sensor (F_{r}). The frequency domain method shows the advantage to use standard and then low cost components, but suffers from a low accuracy. If from a theoretical point of view, the smallest value of the spectral resolution is given by the Fourier transform [2], the situation is significantly different when considering the experimental conditions. In fact, the transition from theory to practical experience inevitably introduces the notion of observation window T_0. Therefore, in a digital system, instead to be considered as infinitesimally small, the spectral resolution ΔF_{f} is constant and defined by $\Delta F_{\mathrm{f}} = 1/T_0$.

This paper aims at exploring an alternative way offering more degrees of freedom, which consists to implement a time-based approach by using the wavelet transform (WT).

II. SAW devices

Surface acoustic wave (SAW) devices, are a key components in communication systems and are widely used as filter, delay lines or resonators. They also offer a news and very promising solutions in a wide range of applications including physical and chemical sensors. In addition to being small, simple and robust, these devices have the advantage of being passive (batteryless), remotely requestable (wireless) and inexpensive if fabricated on a large scale. The use of SAW devices as passive and wireless sensors allows them to operate in extreme conditions such as those with high levels of radiation, high temperatures up to $1000°$C, or electromagnetic interference, in which no other wireless sensor can operate. The SAW devices are highly sensitive to external physical parameters and to any disturbance that may affect the velocity, distance travel or even the mode of wave propagation. A disturbance resulting in a variation of the electrical response of the device (frequency, phase, amplitude...). SAW systems are no exception to this rule and are sensitive to three major types of disturbances: the change in temperature, deformation and in gaseous, liquid or solid species deposited on acoustic wave travel surface. The change in temperature and deformation induces an effect on both a variation of speed (alteration of elastic and piezoelectric coefficients) and a change of path length. Due to their small size, SAW devices can react very fast to the changes in the environmental conditions. Two configurations, can be considered to use SAW devices as wireless sensor: delay line or resonator. In this paper we will focus our study on the resonator one. Its principle as sensor is described in following. The technological advances in the field of SAW have allowed the achievement of resonators with a high quality factor (Q) which allows design sensor based SAW resonator (SAWR) with high sensitivity, accuracy, long-term stability and the possibility of storing electromagnetic energy. A SAWR consists of a piezoelectric substrate, an interdigital transducer (IDT), and two reflectors in the direction of the propagating wave (Fig. 1). The IDT is connected to an antenna. It receives energy for the excitation of the SAW by an electromagnetic wave coming from the interrogation unit. The IDT converts electrical energy to mechanical energy of the surface acoustic wave. The two reflector gratings form a resonating cavity in which a standing wave is generated in the case of resonance. A portion of the stimulating electromagnetic energy is stored in this standing wave. After the stimulating signal is switched off, energy still is present in the form of the SAW. The IDT converts a portion of the mechanical energy back to electrical energy now because the process of energy conversion is partially reversible. The electrical energy is transmitted as an electromagnetic wave back to the interrogation unit and can be analyzed. Typical response obtained at the input of interrogation unit is shown in figure 1. Duration of oscillations is of course depending on quality factor of the resonator but also on the operating frequency. Resonance frequency of the sensor, directly linked to the mesurand, could be extracted from using different techniques.

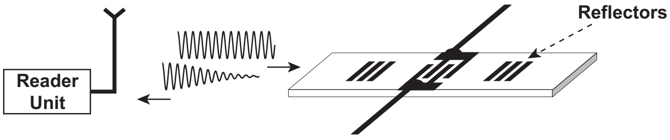

Fig. 1. SAW wireless sensor in one port resonator configuration.

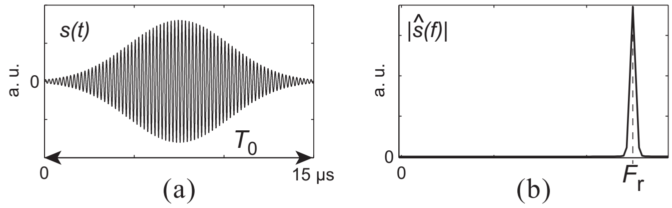

Fig. 2. SAW resonator signal model: (a) time domain, (b) frequency domain.

III. Wavelet transform

If the Fourier transform has the ability to provide the frequency composition of a signal, so what about the pertinence of imaginary exponentials to describe it? What are the acceptable consequences in the sense of Gabor-Heisenberg? (even if the short-time Fourier transform improves the situation, it is not relevant within the meaning of the time-frequency paving, nor in terms of computation time and even sometimes of precision).

Wavelets provide satisfactory answers to all asked questions. They achieve the minimum limit imposed by the uncertainty principle of Gabor-Heisenberg with a better suited time-frequency paving [4]. They also allow considering unimagined analysis ways by using functions families well localized in time and with varied morphological characteristics.

The wavelet transform provides a new kind of analysis because it explores structures as finely as possible at various observation scales to find the one that is most relevant to identify the useful information. It is defined by the weighted scalar product

$$\mathcal{W}f_{s,b} = \frac{1}{\sqrt{s}}\int_{\mathbb{R}} f(t)\psi\left(\frac{t-b}{s}\right)\mathrm{d}t. \tag{1}$$

The operation is therefore to measure the behavior of the signal to be analyzed f around time b and in an observation radius proportional to the scale s. The wavelet transform acts as an operator that breaks up quantity to be analyzed by a family of translated and dilated wavelets $\frac{1}{\sqrt{s}}\psi\left(\frac{t-b}{s}\right)$ from a mother wavelet ψ checking the following properties:

- the mother wavelet ψ is characterized by a number of vanishing moments m such as $\langle t^m, \psi(t)\rangle = \int_{\mathbb{R}} t^m\psi(t)\mathrm{d}t = 0$. This property is decisive to optimize detection of a singularity since in such a case the analysis is unable to detect any polynomial of degree less than $m-1$.
- the wavelet transform also has the ability to reconstruct the signal f from the decomposition coefficients by $C_\psi^{-1}\int_{\mathbb{R}}\int_{\mathbb{R}} s^{-2}\ \mathcal{W}f_{s,b}\ \psi\left(\frac{t-b}{s}\right)\mathrm{d}s\ \mathrm{d}b$ subject to $C_\psi = 2\pi\int_{\mathbb{R}}\frac{|\widehat{\psi}(\omega)|^2}{|\omega|}\mathrm{d}\omega < +\infty$, which is checked if one respects the admissibility condition: $\int_0^{+\infty}\frac{|\widehat{\psi}(\omega)|^2}{|\omega|}\mathrm{d}\omega = \int_{-\infty}^0\frac{|\widehat{\psi}(\omega)|^2}{|\omega|}\mathrm{d}\omega < +\infty$ or also that $\int_{\mathbb{R}}\psi(t)\mathrm{d}t = 0$ et $\int\|\psi(t)\|^2 = 1$ o $\psi \in L^2(\mathbb{R})$.

IV. Frequency measurement: Fourier approach

In the case of SAW resonators, signal to be processed can be modeled by an amplitude modulated gaussian with a carrier frequency F_r (Fig. 2).

Consequently, in theory, it is obvious that the smallest value in terms of spectral resolution is given by the Fourier transform since the spectral support of the analyzing basis is infinitely small and therefore the accuracy tends to the maximum possible. This transform is the ideal tool to reveal the spectral information of a signal as long as it is stationary. However, if we consider experimental signal, its acquisition amounts to multiplie it by a rectangular window of width T_0. Thus, in a digital system, the spectral resolution ΔF_f, which was previously infinitely small, becomes equal to a constant only dependent on the duration of the acquisition window and can be written:

$$\Delta F_\mathrm{f} = \frac{1}{T_0}. \tag{2}$$

It is important to note that the spectral resolution ΔF_f is independent of the number of samples N but also to the sampling frequency F_s. For proof, just observe the relationship

$$T_0 = N \cdot T_\mathrm{s}, \tag{3}$$

and see if N increases then T_s decreases to maintain the constant value of T_0. Thus, for more precision in a Fourier transform, we must increase the observation duration T_0 and therefore the analysis time.

A first improvement way would be to use techniques such as zero-padding, symmetrization, periodization or expansion. In all cases, it comes to artificially increase T_0. We did not used them because they all introduce more or less important artifacts.

Another solution with more degrees of freedom is to implement a time-based approach by the wavelet transform.

V. Frequency measurement: wavelet approach

A. Wavelet-based method

The wavelet transform is a tool particularly well-suited to the detection of singularities in a signal. These ones can be highlighted and characterized by calculating the Lipschitz exponent (or Hölder) and the use of wavelet transform modulus maxima [3]. Our method is precisely based on the calculation of these local modulus maxima but in an original way by deducing the oscillation frequency of the SAW resonator from their recurrence period.

If $\mathcal{W}f(s,x)$ is the wavelet transform of a function $f(x)$ then a modulus maximum is the point (s_0, x_0) where

$$|\mathcal{W}f(s_0, x)| < |\mathcal{W}f(s_0, x_0)| \tag{4}$$

when x belongs to either the right or the left neighborhood of x_0, and

$$|\mathcal{W}f(s_0, x)| \leqslant |\mathcal{W}f(s_0, x_0)| \tag{5}$$

when x belongs to the opposite neighborhood of x_0.

A maxima line is a connected curve of the modulus maxima in the scale space (s, x).

For the considered signal class, and by selecting an appropriate wavelet, computation of maxima lines brings up an alignment of these ones through scales as shown in Fig. 3c. By choosing the most appropriate scales range Δs_{opt}, we can deduce, for a signal with p periods, the expression of F_r by

$$F_\mathrm{r} = \frac{1}{2(2p-1)} \sum_{i=1}^{2p-1} \frac{1}{(t_{s_{\mathrm{opt}}, \|\mathrm{m}_{i+1}\|} - t_{s_{\mathrm{opt}}, \|\mathrm{m}_i\|})}, \tag{6}$$

with $t_{s_{\mathrm{opt}}, \|\mathrm{m}_i\|}$: time point at the optimal scale s_{opt} of the modulus maximum $\|\mathrm{m}_i\|$.

Finally, the use of the wavelet transform leads to a simple calculation of half-periods from which we deduce the center frequency value of the signal. Its main advantage is to stay in the time space and to obtain, with a constant observation duration, the spectral resolution ΔF_w directly related to the sampling frequency F_s and hence to the number of samples N. Thus, the spectral resolution is calculated from the difference between two time points:

$$\Delta F_\mathrm{w} = F_\mathrm{r} - \frac{1}{T_\mathrm{s} + T_\mathrm{r}}, \tag{7}$$

with $T_\mathrm{r} = 1/F_\mathrm{r}$ and $T_\mathrm{s} = 1/F_\mathrm{s}$.

We study in Section V-C the behavior of ΔF_w according to T_0 and N.

B. Wavelet choice

The sensitive point, recurrent to any application, concerns the choice of the wavelet. Indeed, if some are behaving remarkably well (Fig. 3c), all wavelets are not in the same case (Fig. 3b).

In order to select the best wavelets for this signal class, we have tested 129 wavelets representing all important families: `daubechies`, `symlet`, `coiflet`, `biorthogonal`, `reverse biorthogonal`, `meyer`, `gauss`, `morlet`, `mexican hat`, β-`splines` and `shannon`. We have applied them to a set of synthetical signals with three reference frequencies ($F_\mathrm{r} = 5$, 7.5 and 10 MHz) and two lengths ($N = 37500$ and $N = 75000$ samples), which are parameters of experimental signals.

Two criteria have allowed classifying these wavelets: accuracy of the frequency F_r and computation speed. The four wavelets that provide best results are: `haar`, `gaus1`, `rbio3.1` and `gaus4`.

We have chosen the wavelet `gaus1` for the implementation of the method.

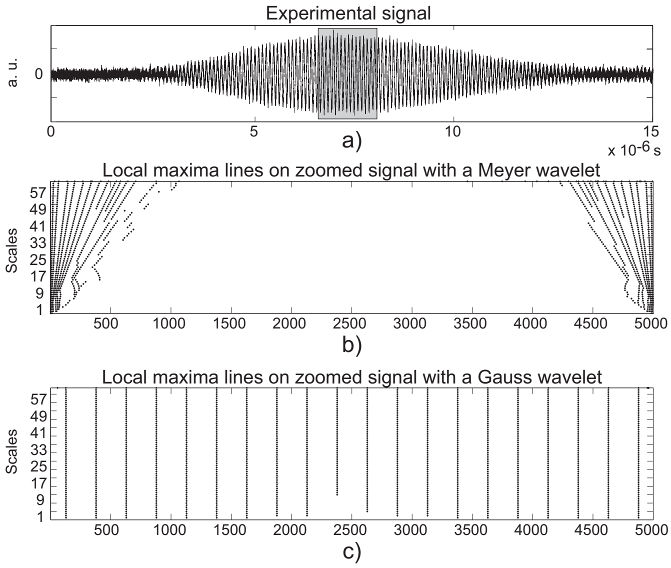

Fig. 3. The analyzed experimental signal and modulus maxima lines of the continuous wavelet transform for: b) a bad wavelet, c) a good wavelet.

C. Method extension

In practice, we have very little flexibility on parameters. Indeed, the observation duration T_0 is imposed by physical characteristics of SAW devices and sampling frequency F_s (hence the number of samples N) is imposed too by the acquisition system

So, by considering T_0 and N as constants, we know from (2) that the Fourier transform provides a constant spectral resolution ΔF_f, which is not the case for the wavelet transform because, as shown in (7), the spectral resolution ΔF_w depends on the reference frequency F_r.

The comparison in terms of performance of the field of use for both resolutions ΔF_f and ΔF_w is obtained by analyzing the intersection point in relationships (2) and (7), and for which we can write

$$F_\mathrm{r} = \frac{\sqrt{4N+1}+1}{2T_0} \tag{8}$$

Thus, if we have the choice of the resonator frequency F_r, it is necessary to choose it smaller than the expression value (8) to obtain a better spectral resolution.

However, if F_r is imposed, we propose to improve ΔF_w by introducing an upsampling factor R leading to the calculation of intermediates samples and obtained by the spline method or the Shannon relationship. These new points are used to artificially increase the sampling frequency F_s. Both techniques giving the same results, we have chosen the Shannon way.

The results of this technique are presented in Fig. 4 for two synthetical signals ($T_0 = 15$ μs, $F_\mathrm{s} = 2.5$ GHz, $N = 37500$ samples): the first with $F_\mathrm{r} = 5$ MHz and the second with $F_\mathrm{r} = 10$ MHz.

Figs. 4a and 4b show that the spectral resolution ΔF_w is independent of the number of periods. Since F_s is fixed, ΔF_w

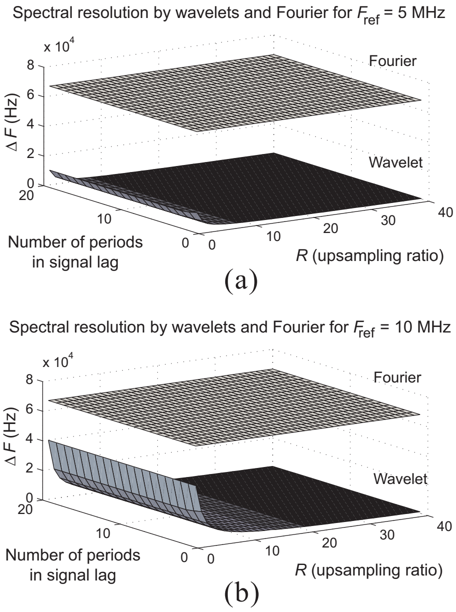

Fig. 4. Spectral resolution and comparison between wavelets and Fourier for two synthetical signals ($T_0 = 15$ μs, $F_s = 2.5$ GHz, $N = 37500$ samples): (a) $F_r = 5$ MHz and (b) $F_r = 10$ MHz.

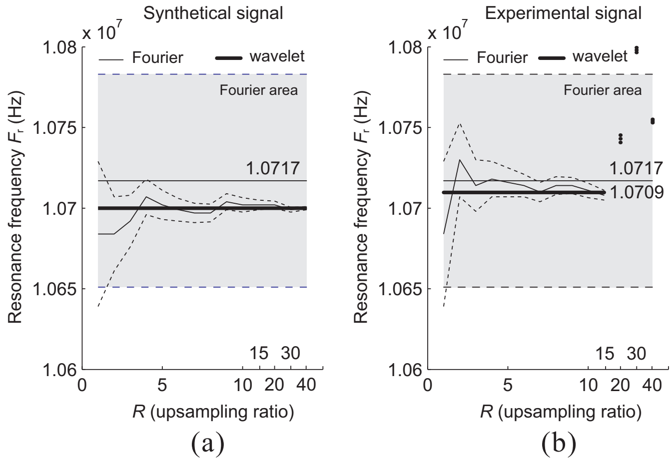

Fig. 5. Resonance frequency measurement for: (a) synthetical signal and (b) experimental signal with $F_r = 10.700$ MHz, $F_s = 2.5$ GHz, $N = 37500$ points, $T_0 = 15$ μs and $p = 8$ periods.

is independent of T_0 which is not the case at all of the Fourier transform. This remark allows us to apply the wavelet-based method on a small signal lag (5 % to 10 %), the important thing being to select few periods. These same figures also show that choosing a low upsampling ratio, spectral resolution is significantly improved.

The introduction of upsampling provides an additional degree of freedom and allows calculating R according to the desired spectral resolution, F_r being fixed or variable. From (3) and (7) we have

$$\Delta F_w = F_r - \frac{1}{\frac{T_0}{RN} + T_r} \tag{9}$$

and finally,

$$R = \frac{(F_r - \Delta F_w) F_r}{\Delta F_w F_s} \tag{10}$$

Thus, for $F_r = 5$ MHz and $T_0 = 15$ μs, we get the following spectral resolutions:

- $\Delta F_f = 66$ kHz by Fourier,
- $\Delta F_w = 10$ kHz by wavelets without upsampling,
- $\Delta F_w = 2$ kHz by wavelets with an upsampling $R = 5$.

VI. Experimental results

We have applied the wavelet-based method on an experimental signal provided by a SAW device (Fig. 3a) and on synthetical signal, both having the following following characteristics: $F_r = 10.700$ MHz, $F_s = 2.5$ GHz, $N = 37500$ samples, $T_0 = 15$ μs and $p = 8$ periods.

The results are presented in Fig. 5. Fig. 5a clearly shows that values of F_r obtained by wavelets are always within the limits range imposed by the spectral resolution and thus for $R = 40$ we get

$$10669 \text{ MHz} \leqslant F_r \leqslant 10701 \text{ MHz} \tag{11}$$

with $\Delta F_w = \pm 1$ kHz while $\Delta F_f = \pm 66$ kHz.

Fig. 5b shows the same results but with a limit since, beyond $R > 15$, values of F_r are false. These values can be explained by the presence of noise on the experimental signal (Fig. 3a), unlike the synthetic signal, for which upsampling generates artifacts in wavelet maxima lines.

VII. Conclusion

In this article, we compared the Fourier transform and the wavelet transform in the case of SAW resonance frequency measurement. If the Fourier transform can be envisaged, wavelet transform possibilities make it an attractive alternative. Indeed, the wide variety of available wavelet bases allows reaching a high level of accuracy and a competitive computation time. We have given necessary conditions of the center frequency F_r, the number of samples N and the observation time T_0 to get a better than Fourier spectral resolution. We also proposed an analytical expression setting the upsampling parameter R according to the desired spectral resolution. Finally, the method has been validated on both synthetic and experimental signals.

References

[1] D. Morgan, *Surface Acoustic Wave Filters With application to communications and signal processing*, 2nd ed., Academic Press, 2007.
[2] M. Hamsch, R. Hoffmann, W. Buff, M. Binhack, and S. Klett, *An Interrogation Unit for Passive Wireless SAW Sensors Based on Fourier Transform*, IEEE Trans. Ultrason., Ferroelectr., Freq. Control, vol. 51, pp. 1449, 2004.
[3] S. Mallat, *A Wavelet Tour of Signal Processing: The Sparse Way*, 3rd ed., Academic Press, pp. 173-186, 2008.
[4] A. Scipioni, P. Rischette, P. Schweitzer and J. Mathieu, *Wavelet-based estimator versus spectral correlation method for acoustic target sizing*, Meas. Sci. Technol., Vol. 20 (11), 115101, 2009.