Target sizing: spectral method versus wavelet-based estimator

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***Abstract*—The problems tackled in this article are those of the sizing characterization of an immersed target. This wire target is insonified by an ultrasonic broadband transducer, and our goal is to compare the diameter estimation by two different methods which both rely on the analysis of the backscattered echo. It is shown that the first part of the backscattered echo (QRBE) contains the size information (diameter) of the target. The first method of radius estimation is based on the identification of the optimal parameter which ensures the best superimposition of the spectral backscattered echo segment on the quasi-rigid form function (QRFF). The second technique relies on a family of synthetic echoes obtained from the resonant scattering theory (RST). Those are analyzed by a wavelet transform of which the coefficients are used to establish a model identifying the experimental signals. Results obtained with copper and steel wires (with wires ranging from 0.25 mm to 1 mm) are compared with experimental measurements. For the two methods, the results are similar for the largest diameters. But the more the diameter decreases, the more the wavelet-based estimator is distinguished by giving a better relative accuracy.**

I. INTRODUCTION

The granulometry of particles in suspension in a liquid, like the measure of the volumetric rate of suspended solids in wastewater, often needs to characterize these particles especially in terms of size. Existing acoustic methods are based on the analysis of the attenuation-spectrum [1]. However, the main disadvantage for all of these solutions is that the material of particles must be known. In this communication, we propose two methods without contact that don't need to know the nature of the particle belonging to a class of material. They rely on the backscattered echo study of a millimetric wire target which is insonified by an ultrasonic broadband transducer.

The first method of radius estimation is based on the identification of the optimal parameter which ensures the best superimposition of the spectral backscattered echo segment on the form function.

The second technique relies on a family of synthetic echoes obtained from the resonant scattering theory (RST). Those are analyzed by a wavelet transform of which the coefficients are used to establish a model identifying the experimental signals.

Outline: Section II points out some essential elements of the RST theory [2] indispensable to a good understanding of the physical phenomena met and the first part of the backscattered echo, called quasi-rigid backscattered echo (QRBE), and the quasi-rigid form function (QRFF) are described. Section III presents the spectral method while Section IV details the time method. In Section V we give the experiment conditions and we present the results. Finally last section compare these two estimating techniques and different extending ways are proposed.

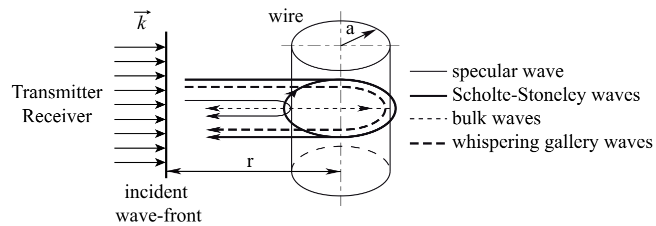

Fig. 1. Nature of the waves crossing a target insonified by an ultrasonic wave.

II. PHYSICS OF THE MEDIUM, QUASI-RIGID BACKSCATTERED ECHO AND FORM FUNCTION

A rectangular impulse exciting a piezoelectric transducer produces an ultrasonic wave which insonificates a homogeneous and solid wire immersed in a liquid. The RST theory allows to efficiently tackle the diffusive phenomena which result from it. As shown in Fig. 1, various waves with specific behaviors form the backscattered echo:

- a first wave group (external surface waves) sensitive to the target geometry and forming the beginning of the echo is the superimposition of the specular wave and the creeping waves of Franz and Scholte-Stoneley [3],
- a second group (internal to the target) is formed by bulk waves, Rayleigh wave and whispering gallery waves.

The methods compared aim at estimating the target diameters whatever is their material. Thus, the study relies on the first group, after extraction by windowing of the quasi-rigid part of the backscattered echo (QRBE).

The form function, so called scattering magnitude, results from the theory of the collisions [4]. It is without unit and

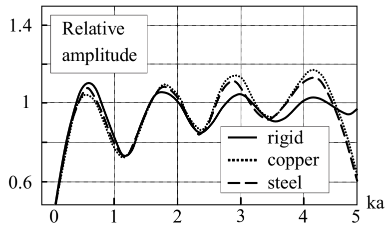

Fig. 2. Quasi-rigid form function for wires.

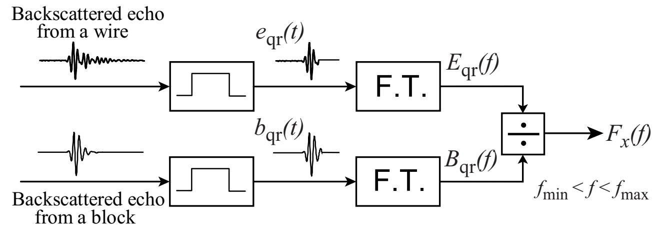

Fig. 3. Schematic diagram to obtain the normalized quasi-rigid echo spectrum $F_x(f)$.

represents the variations of magnitude of the wave scattered according to the observation angle θ and to the size a of the target compared to the wavelength ka.

The quasi-rigid backscattered pressure of an insonified wire [5] is writing

$$P_{s,qr}(ka) = P_i(ka)\sqrt{\frac{a}{2r}} f_{qr}(ka) e^{2jkr}, \quad (1)$$

with

- $P_{s,qr}(ka)$: Fourier transform of the quasi-rigid backscattered echo,
- $P_i(ka)$: Fourier transform of the incident ultrasonic wave,
- $f_{qr}(ka)$: quasi-rigid backscattered form function of the target,
- a: radius of the wire,
- r: distance from the origin to the field point.

Fig. 2 shows the results for a rigid wire and for wires made of copper and steel [6].

The good coincidence of the extrema of the quasi-rigid form functions for wire targets in the interval of $ka \in [0.5, 3]$ confirms their slight dependence with respect to the physical nature of the material. These functions are similar to the function obtained for a rigid target. This last one noted $F(ka)$ is the reference function we use for estimating the radius of a target in the following parts using the spectral correlation method.

III. Spectral method

We only study the QRBE ($e_{\mathrm{qr}}(t)$) resulting from the time gating of the backscattered echo. The time gate is placed at the beginning of the echo with a duration equal to the incident pulse. Then we calculate the spectrum of the quasi-rigid part of the ultrasonic echo by a short time Fourier transform. This spectrum is next normalized by division of the spectrum of the gated echo backscattered from a block simulating an half space ($b_{\mathrm{qr}}(t)$). Thus, we obtain a segment of form function (function of reference $F(ka)$) limited to the bandwidth of the ultrasonic transducer $[f_{min}, f_{max}]$.

This procedure is summarized by the diagram in Fig. 3.

At last, the target size estimation is done by looking for the optimal repositioning of the normalized backscattered quasi-rigid echo $F_x(f)$ on the reference form function $F(ka)$.

For estimating the radius a of the wire, we have to find its optimal size for which the superimposition of the vector $F_x(f)$ on the form function of reference $F(ka)$ is the best.

For a frequency in the system bandwidth, the process requires the following three steps :

- by homothethy from the form function of reference $F(ka)$, we compose a matrix of N functions by considering N size a_N with a step Δa corresponding to the desired precision,
- we determine the vector of correlations between the columns of the matrix and the column of a vector formed by the segment values,
- the highest correlation coefficient between the segment and the function will be the criterion of optimal superimposition and gives us the optimal value which estimates the radius a of the wire under test.

IV. Wavelet-based method

When we observe the morphology of a backscattered echo, we think unquestionably about the one of some wavelets. This likeness gave us the idea to use wavelets as a characterization tool of this class of signals in order to develop another method for the estimation of wire target diameter. However, their great diversity necessarily puts the question of the best family choice well-adapted to the following problem [7], [8]:

- to find the constant relevant scale where wavelet coefficients are maximum and where the relation of order which links them to the different diameters exists for all the material belonging to a same class.

The superimposition of the Scholte-Stoneley waves with the specular wave varies according to the target diameter, which directly involves a variation of the QRBE width. Thus, it is necessary to seek a stable zone of measurement independent of various diameters. Consequently we must select the size of the segment and its optimal position. For reasons of calculation costs and real time application, we have opted to take the shortest of these segments (10 %). Moreover we have chosen the earliest position at the beginning of the echo (0.5 %) because this one comprises on the one hand, information of wire dimension, and on the other hand is less sensitive to possible residual waves (bulk waves and whispering gallery waves).

Among tested wavelets, the most robust, i.e. the one which respects the more often a monotony relation between diameters and maxima of the coefficients, is the wavelet β-spline inverse 5/3 (denoted as 'rbio2.2'). We must emphasize that discrete wavelet transform has been tested but results was not good

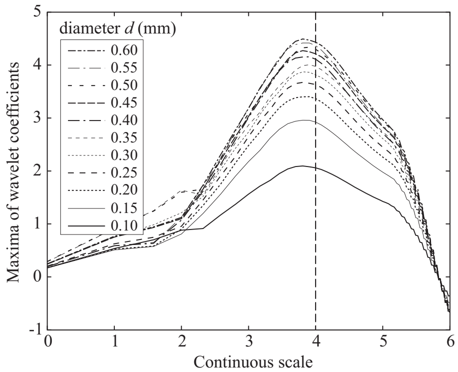

Fig. 4. Maxima of the wavelet coefficients versus continuous scale for the wavelet rbio2.2.

enough. That is why the continuous wavelet transform with the 'rbio2.2' wavelet (CWT-rbio2.2) has been used in the rest of this work.

The main disadvantage of a continuous wavelet transform concerns the computing time which is much higher than in a discrete approach. Thus, we propose an intermediate process which consists in applying CWT-rbio2.2 by using the mirroring technique [9] in order to minimize the edge effects but by analyzing the integer and dyadic scales. The results obtained are presented in Fig. 4. One can observe that not only maxima appear around only one scale (here 4) but moreover the relation of order is fully respected.

As indicated in [10], it is possible from the theoretical form function to generate synthetic backscattered echoes of wire corresponding to given diameters. By this way we create several backscattered echoes for diameters d (mm) $\in \{0.10, 0.15, 0.20, 0.25, \cdots, 0.6\}$ which will be used as reference in the development of the model.

By minimizing the error in the sense of least-squares of a polynomial approximation relating to the maxima obtained at scale 4 according to diameters, we obtain the expression of the model being used as reference for the searched out estimator:

$$d = 10^{0.014c_{\rm m}^3 - 0.048c_{\rm m}^2 + 0.181c_{\rm m} - 1.291} \quad (2)$$

where d means diameter and $c_{\rm m}$: maxima of wavelet coefficient.

Lastly, after the same processing on the experimental backscattered echoes, it is easy to identify the diameter corresponding to the target by using (2).

V. Experimentation and Results

Experiments were carried out with wires vertically imbedded in a tank filled with water at ambient temperature. The experimental set up is shown in Fig. 5 and the parameters summarized in Table I. Wires are fixed on a plexiglass framework in the axial beam of an ultrasonic transducer and a block of aluminium is used to calibrate the system.

The transmitter/receiver is a planar wideband transducer (-6 dB bandwidth: 0.8 MHz - 1.4 MHz) with a diameter

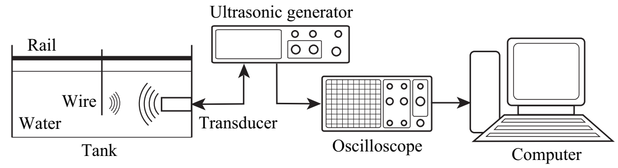

Fig. 5. Experimental setup.

TABLE I
Parameters of experimentation.

Frequency bandwith (kHz)	Radii range a (mm)
$f_{min} = 800$, $f_{max} = 1400$	from 0.05 to 0.75
$\Delta f = 6$	$\Delta a = 5.10^{-3}$
Discrete frequencies f_i (kHz)	Discrete radii a_j (mm)
$f_i = 800 + 6i$	$a_j = 0.05 + 5.10^{-3} j$
$0 \leqslant i \leqslant 100$	$0 \leqslant j \leqslant 40$

of 0.95 cm. A generator Ultimo 2000 allows to generate a negative ultrasound pulse and to amplify the signals. The receiver gain can vary from -10 dB to 80 dB, the impulse width can be adjusted from 25 ns to 1000 ns. Echoes are acquired by a digital oscilloscope Tektronix TDS 320 with a bandwidth of the digital acquisition system of 100 MHz, and a sampling rate of 20 MS/s. The data are next acquired on a computer by an interface IEEE 488.2.

A. Results with the spectral method

Measurements are carried out in order to estimate three different wire diameters made of steel and copper. On Fig. 6 we can see the optimal superimposition of the vector $\boldsymbol{F}$ on the quasi-rigid form function $F(ka)$ for a steel wire of diameter 0.5 mm.

Results are given in Table II. For the high radius, the estimation presents a good accuracy but deteriorates when radius is lower than 0.25 mm. In reality, segment $\boldsymbol{F}$ approaches a line and the method of correlation analysis is less adapted for repositioning this type of curve. A solution is to increase the bandwidth of the transducer for instance or to complete the correlation with a least-squares analysis.

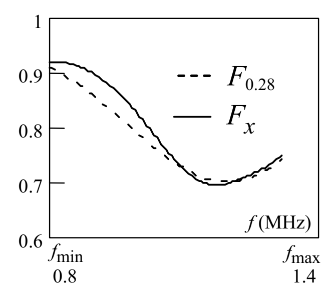

Fig. 6. Superimposition of the segment on the form function of a steel wire: $a = 0.250$ mm.

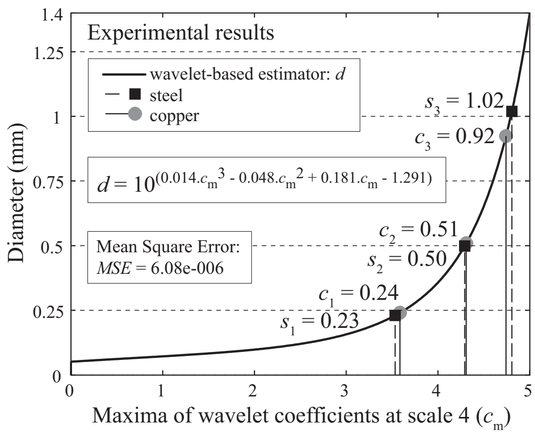

Fig. 7. Wavelet-based estimator: representation of the law existing between the diameter and the wavelet coefficients. Validation of the estimator for steel and copper echoes.

TABLE II
COMPARATIVE RESULTS OF THE TWO METHODS FOR STEEL AND COPPER WIRES OF DIAMETER 0.25, 0.50 AND 1.00 MM.

		Steel		Copper	
Reference diameter	Computation methods	SCM[1]	WM[2]	SCM	WM
∅ 0,25 mm	Estimated value	0,31	0,23	0,32	0,24
	Relative error	24%	8%	28%	4%
∅ 0,50 mm	Estimated value	0,56	0,50	0,56	0,51
	Relative error	12%	0%	12%	2%
∅ 1,00 mm	Estimated value	1,08	1,02	1,12	0,92
	Relative error	8%	2%	12%	8%

[1]Spectral Correlation Method, [2]Wavelet-based Method.

B. Results with the wavelet-based method

First of all, we apply a normalization step on the experimental echoes which are the same as those used in the correlative method. To this end, we calculate the magnitude difference ratio between theoretical and experimental echoes for the measurement point corresponding to $d = 0.4$ mm. The goal is to place these two echoes in the same magnitude range.

Then mirroring procedure is implemented on the same signal parts as those chosen for theoretical echoes. As a result, we can apply CWT-rbio2.2 and the maximum of the coefficients at scale 4 is submitted to the estimator.

Fig. 7 presents the results and we can note that this method allows one to obtain good estimates of various diameters. By the way, we want to emphasize the good estimate of the 1 mm diameter though its value is outside the range used for the model construction.

VI. COMPARISON AND CONCLUSION

Even if in the two cases, the second processing step is based on the reference model development, the two methods are quite different one from the other when we look at the principle: the first tackles sizing problem by employing the spectral domain while the second makes it by considering the time domain. Thus in the first method, quasi-rigid backscattered echo to be analyzed is compared with quasi-rigid form function and in the second case, wavelet maxima are submitted to a model built from theoretical quasi-rigid backscattered echoes.

Steel and copper wires have been used to compare the two methods, with diameters ranging from 0.25 mm to 1 mm. Table II presents comparison results and shows that in the two cases, estimations are of rather good quality in terms of precision. Nevertheless, we must highlight that the wavelet method provides better results than the correlative method and this, for all the diameters measured. Furthermore in the two methods, the relative error increases when the diameter decreases. In the correlative method, the relative precision varies between 8% and 28% and when the diameter decreases, it tends to deteriorate. By increasing the central frequency of the transducer, we can provide an improvement of the precision for the small diameters, but at the price of a diameter range diminution ($ka < 3.5$). In the wavelet-based method, the relative precision is quite stable. It evolves between 0% and 8% and does not seem to be sensitive to the reference diameter value.

These two sizing methods can also be extended to spherical targets and complementary studies are in hand aiming at extend it to other materials belonging to the same class and for a wider range of diameters.

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