Design and computation of laminated composite structures

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Abstract

The design and computation of laminated composite structures in terms of their strength is a complex task because many different mechanisms are involved in the damage and rupture to which these materials are subject. The great diversity of the damage mechanisms and their patterns of evolution make it extremely difficult to estimate the strength margins. In the case of woven plies laminates, the number of damage mechanisms is fairly small (no transverse rupture occurs and the material has a greater resistance to delamination) and the behaviour of the material is fairly simple to model up to rupture. The strategy adopted in this study on designing laminated composite structures is based on the choice of materials (woven or unidirectional plies) and the use of simple models (focusing on the rupture of the first ply) with which industrial structure analyses can be carried out. The choice of materials does not depend only on mechanical criteria but also on the simplicity of the corresponding modelling procedures.

Keywords: A. Textile composites, B. Mechanical properties, B. Fracture, C. Damage mechanics, C. Laminates.

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1. Introduction

The qualities for which laminated composites are mainly chosen are their specific rigidity and strength. Although designing structures according to rigidity does not raise any particular problems, designing with respect to strength is much more complicated. When these materials are subjected to severe loading up to rupture, the many mechanisms responsible for the damage and rupture occur on different scales: matrix micro-cracking, fibre/matrix debonding, transverse rupture, fibre rupture, delamination, rupture of the plies and the laminate [1]. The great diversity of the damage mechanisms and their patterns of evolution make it extremely difficult to estimate the strength margins.

When transverse rupture and delamination occur in the edge zones of unidirectional (UD) ply laminates, they will extend inside the structure and it is therefore not yet possible (by performing 3-D modelling) to simulate the damage processes occurring in complex industrial structures. In the case of woven plies laminates, the number of damage mechanisms is fairly small (no transverse rupture occurs and the material has a greater resistance to delamination) and the behaviour of the material is fairly simple to model up to rupture [2]. Moreover, in the case of woven ply laminates, the rupture of a ply is associated to fiber rupture and is very close to the rupture of the laminate and the structure. This observation noted in experiments, at least for static loadings, simplifies simulation until rupture since it is not necessary to simulate propagations of cracks in the plies. A finite element calculation in terms of plane stresses until the rupture of the first ply and which includes the elastic plastic damage behaviour of the woven ply enables to describe the rupture of a structure [3].

As we already specified, the principal difficulties in the case of the laminated composites relate to the modelling and the simulation of propagations of cracks in the plies and interfaces (delaminations). For studying UD ply laminates under static and fatigue loading, a first ply failure model was previously proposed for structures not subject to delamination [4]. Within this restricted framework, a model based on plane stresses accounting for the elastic plastic damage behaviour of the ply again turned out to be sufficient. This type of model can be used for dealing with tubes, for example, in contexts where a high level of safety is required.

Obviously, the use of UD plies is optimal when rigidity plays a dominating part. Due to the reduction in the costs of new high-modulus fibres (pitch fibre), it seems logical to use this type of fibre for applications where rigidity is of particular importance (in the case of structures subjected to dynamic and buckling loading, for instance). For these applications, it suffices to use the elastic behaviour to be able to predict the rigidity. However, these fibres have very weak characteristics in compression and particular precautions are to be taken in order to know them as well as possible.

The strategy adopted in this study on designing laminated composite structures is based on the choice of materials (woven or UD plies, high modulus fibres) and the use of simple models (focusing on the rupture of the first ply) with which industrial structure analyses can be carried out. It is proposed here to study the following three cases, using this strategy:

(i) In the case of woven plies laminates, the behaviour of the material is fairly simple to model and the reliability of the simulation up to rupture is improved. The woven ply model is recalled and finite element computations are compared with experiments in the case of a laminated composites plate containing an open hole subjected to tensile loadings.

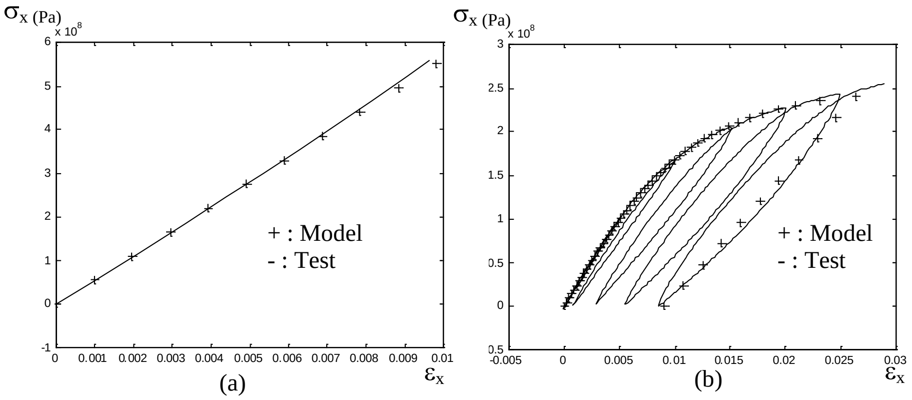
(ii) In the case of UD ply laminates not subject to delamination under static and fatigue loading, a first ply failure model turned out to be sufficient. The main features of the model are pointed out and some experimental results are presented.

(iii) High modulus fibres are suitable for applications where rigidity is of particular importance (dynamic and buckling). However, these fibres are not very resistant to compression and their response to high compression conditions therefore needs to be precisely determined. A pure bending test is described and some experimental results showing the nonlinear behaviour in the fibre direction are presented.

2. Woven ply laminates

Woven ply laminates have weaker mechanical characteristics and are more expensive than UD ply laminates. However, they are used in industry (to make helicopter blade skins, for example). The explanation often given by companies for this choice of woven ply laminates is that they are not subject to transverse rupture, which can be catastrophic in the case of UD plies. Moreover, these materials are more resistant to delamination. The number of damage mechanisms liable to occur is thus reduced and the behaviour of the material is simpler to model up to rupture.

2.1 Damage behaviour of woven plies



(a) (b)

Fig. 1. Tensile tests on a $[0]_8$ (a) and $[45]_8$ (b) woven ply laminates.

The behaviour of materials of this type and the modelling procedure used were previously described in Ref. [2]. The material is reinforced with a carbon fabric of the four-harness satin type, with balanced warp and fill yarns. In the fibre directions, the woven ply shows a brittle linear elastic behaviour when subjected to tension (see Fig. 1(a)). The damage occurring in these directions does not affect the behaviour of the ply under traction loading. When shear loading was applied (from a tensile test on a $[45]_n$

laminate), a decrease in the shear modulus as well as inelastic strains were observed (see Fig. 1(b)). The decrease in the modulus was due to the ply shear stress, which generated some fibre/matrix decohesion and matrix micro-cracks within the warp and fill yarns. The inelastic strains and the loading-unloading hysteresis observed (Figure 1(b)) were mainly due to the slipping/friction processes occurring between the fibres and matrix as the result of the damage.

With a view to modelling the behaviour of laminates with a woven reinforcement, we have adapted the “meso-scale” model developed for unidirectional plies proposed by Ladevèze [5-6]. This model was designed for dealing with woven plies with balanced or non-balanced warp and fill yarns. The damage kinematics adopted was based on the following three internal damage variables (d_1, d_2, d_{12}): the brittle fracture of fibres in the warp and fill directions and the decreasing stiffness under shear loading, respectively. The gradual development of the damage d_{12} depends on the shear load as well as on the traction load, which generates micro-cracks in both the fill and the warp components. These micro-cracks, which are mainly located at the fibre/matrix interfaces, are assumed to run parallel to the fill and warp directions. Under the assumption of plane stresses and small perturbations, we can write the strain energy of the woven ply in the following form:

$$E_D^{ps} = \frac{1}{2}\left[\frac{\langle\sigma_1\rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle\sigma_1\rangle_-^2}{E_1^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\langle\sigma_2\rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle\sigma_2\rangle_-^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})}\right] \tag{1}$$

where $< . >_+$ is the positive part and $< . >_-$ is the negative part. The tension energy and compression energy are split in order to describe the unilateral feature due to the opening and closing of the micro-defects. From this potential, thermodynamic forces associated with the tension and shear internal variables d_i (i=1 and 2) and d_{12} are defined:

$$Y_{d_i} = \frac{\partial E_D^{ps}}{\partial d_i} = \frac{\langle\sigma_i\rangle_+^2}{2E_i^0(1-d_i)^2}; \qquad Y_{d_{12}} = \frac{\partial E_D^{ps}}{\partial d_{12}} = \frac{\sigma_{12}^2}{2G_{12}^0(1-d_{12})^2} \tag{2}$$

The development of the internal variables depends on these thermodynamic forces and more precisely on their maximum values during the history of the loading. In traction, the development of d_1 and d_2 is brutal in order to represent the brittle behaviours according to the warp and fill directions. To take into account the traction/shear coupling during the development of d_{12}, we define the equivalent thermodynamic force and the maximum value of this force during the history of the loading:

$$Y = \alpha_1 Y_{d_1} + \alpha_2 Y_{d_2} + Y_{d_{12}} \qquad \textit{and} \qquad \underline{Y}(t) = \sup_{\tau \le t}(Y(\tau)) \tag{3}$$

where α_1 and α_2 are the tension/shear coupling coefficients. It should be noted that this equivalent force, which govern the development of the progressive damage variable d_{12}, does not depend on the compression stresses in the warp and fill directions. As for the unidirectional plies [6], a linear law with respect to the square root of $\underline{Y}$ is chosen to describe the damage variable development:

$$d_{12} = \left\langle \frac{\sqrt{\underline{Y}} - \sqrt{Y_O}}{\sqrt{Y_c} - \sqrt{Y_O}} \right\rangle_+ , \quad d_1 = d_2 = 0 \ \text{ if } (d_{12} < 1 \text{ and } Y_{d1} < Y_{1f} \text{ and } Y_{d2} < Y_{2f}) \ \text{ else } \ d_{12} = d_1 = d_2 = 1 \tag{4}$$

where the constant parameters Y_o and Y_c correspond to the threshold and the critical value of the development of d_{12} (which varies from 0 to 1). Y_{1f} and Y_{2f} are the parameters, which define the ultimate forces in the warp and fill directions. After loading on a laminate $[45]_8$ (Fig. 1(b)), inelastic strains are observed. These strains can be linked to the slipping/friction phenomena between the fibres and matrix as a consequence of the damage. Because of the warp and fill fibre directions, which prevent traction inelastic strains, only the shear inelastic strains are significant. We shall describe these strains by a plastic hardening model [2]. The loading-unloading hysteresis, which is mainly due to the slipping/friction phenomena between the fibres and matrix, is not modelled. The high resistance of woven ply laminates to delamination, in addition to the fact that they are not subject to transverse rupture, allows to simulate the behaviour up to rupture of complex structures made of these materials. For this purpose, a simple finite element model of the shell type, based on plane stresses and accounting for the inelastic damage behaviour of the woven ply, can be used. This relatively simple modelling procedure is not suitable in the case of UD plies, in which transverse rupture and delamination processes resulting from the 3-D effects mentioned above are factors responsible for the rupture of the laminate.

2.2 Behaviour of a perforated plate subjected to tension

In order to test the model, comparisons were made between the results of an experimental test and those of a simulation involving a perforated plate under traction loading. The perforated plate (Figure 2) was a $[45]_4$ laminate made of woven plies. Figure 3 makes it possible to compare the measured strain (gauges g1, g2 and g3, Figure 2) with the simulated strain (the mean strain on the surface of the gauge). It can be seen here that the model accurately predicted the breaking force and the strongly non-linear behaviour. The simulation predicted very high levels of damage (more than 0.5). A further study is now being carried out in order to measure the damage levels involved [7]. Comparisons with other stacking sequences are shown Figure 4. For the quasi-isotropic laminate ($[22.5°,-22.5°]_s$) it was observed that the simulation under predicted the experimental result (rupture in the fibre direction). Currently, we are looking to introducing an internal length in order to better predict the fibre direction rupture in the case of structures with high gradient stresses.

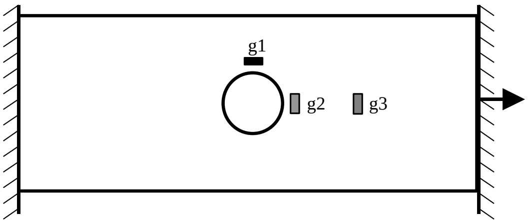


Fig. 2. Traction test on a perforated plate made of $[45]_4$ woven ply laminate.

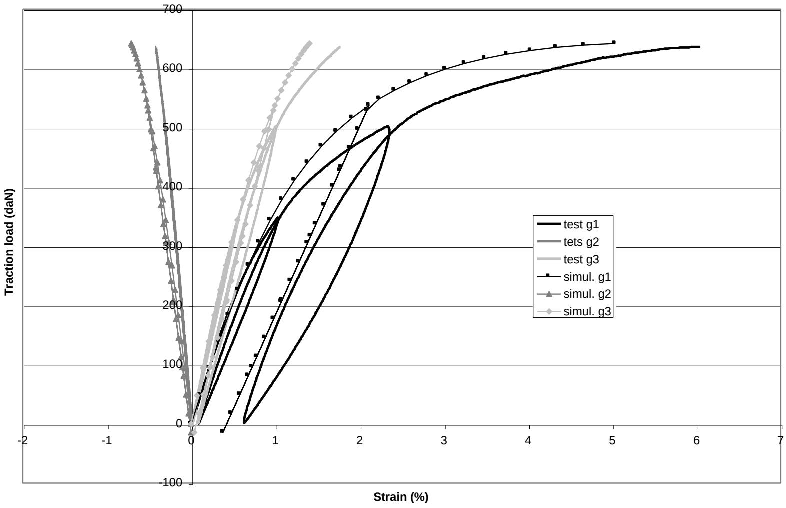


Fig. 3. Measured and simulated strains (see Figure 2).

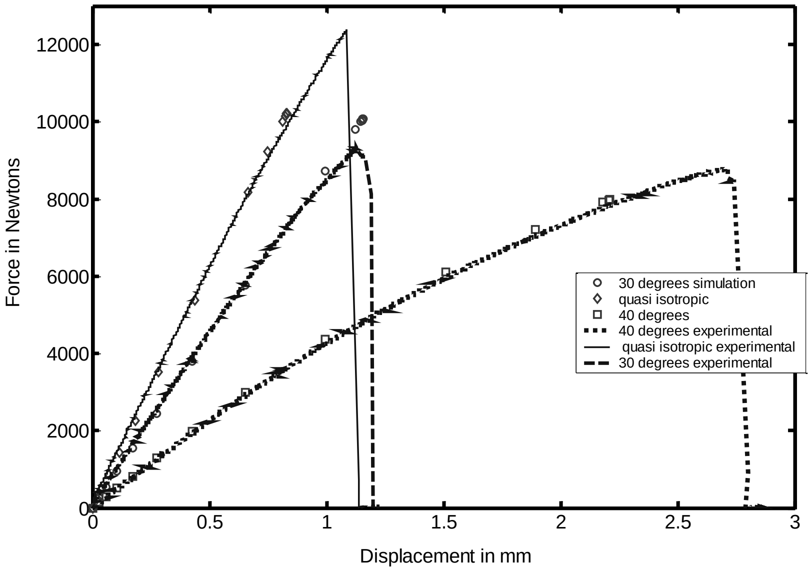


Fig. 4. Testing and simulation for perforated plates in traction.

3. Fatigue behaviour

For dealing with UD ply laminates, it was proposed to model the material up to rupture of the first ply in the case of structures or laminates which are not subject to delamination, under static and fatigue loading conditions [4]. Within this restricted framework, a model based on plane stresses accounting for the elastic plastic damage behaviour of the ply again turned out to be sufficient.

3.1 Behaviour of a unidirectional ply under static and fatigue loading.

The model previously presented [4] is a non-linear cumulative damage model, enables to describe the development of damage under both static and fatigue loading conditions. The delamination processes are not accounted for in this model, which is therefore relevant only to some particular structures and laminates that are not subject to delamination (those which have no edges, as in the case of tubes, for example). The validity scope of the model described here depends on the ‘diffuse damage’ phase (which is associated with micro-cracks) up to the first intra-laminar macro-crack only. In this respect, the modelling procedure adopted differs from the approaches that describe the macro-crack density [8,9]. This type of model can be used in contexts where a high level of safety is required.

The material is assumed to be brittle and non-sensitive to the cyclic loading occurring in the direction of the fibres. The in-plane transversal and shear moduli are modified under the assumption that a gradual damage process is involved, via the micro-cracks running parallel to the fibres. The development of the damage depends on the maximum static and cyclic loads and their amplitude as well as on the level of damage involved. Furthermore, it uses the cumulative damage where damage variables (for shear and transverse tension) are obtained by addition of two terms: one part is due to static loading and the other one is governed by fatigue loading. The static damage evolution is linear versus associated equivalent force. For fatigue damage evolution, experiments have shown its double dependency on maximun load and amplitude load; so a law which respects this observation has been built:

$$\frac{\partial d_f}{\partial N} = c * \left\langle \left(\Delta\sqrt{Y}\right)^{\alpha} * \left(\sqrt{Y_{\max}}\right)^{\beta} - Y_{0f} \right\rangle_+ \qquad \Delta\sqrt{\mathrm{Y}} = \left\{ \sup_{\tau \in cycle(t)} - \inf_{\tau \in cycle(t)} \right\} (\sqrt{\mathrm{Y}}) \qquad Y_{\max}(t) = \sup_{\tau \in cycle(t)} (Y) \tag{5}$$

where the constant parameter Y_{of} corresponds to the threshold in fatigue.

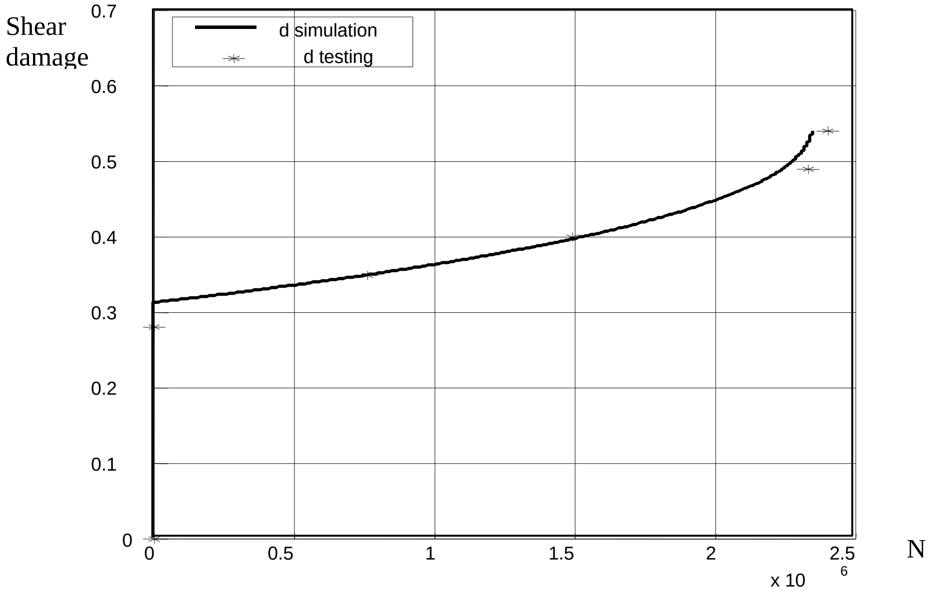


Fig. 5. Testing and simulation of damage evolution

(fatigue loading, σmax=120MPa, stress ratio R=0.5).

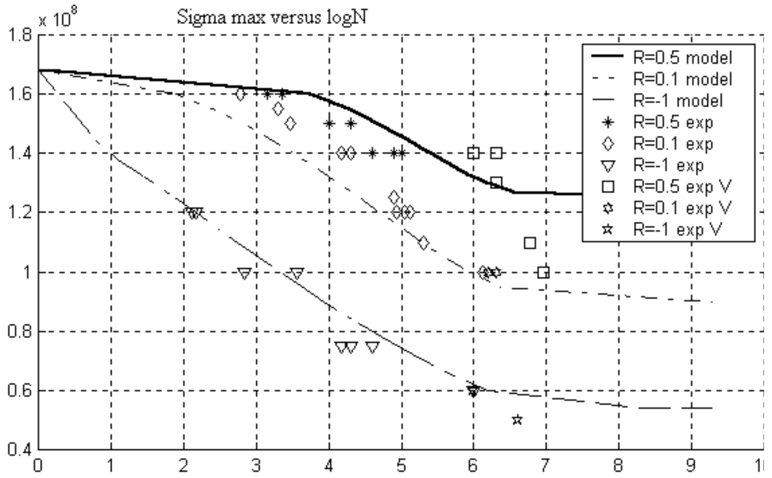


Fig. 6. Fatigue behaviour on $[\pm45]_{ns}$ laminates; the notation “exp V” corresponds to specimens which did not break (tests from Petermann [10]).

An example of the evolution of the damage process in the case of a tension/ tension fatigue test on a $[\pm45°]_{ns}$ laminate is shown in Figure 5. The first level of damage (d=0.3) was found to occur during the first cycle. The damage then increases slowly at each cycle, and the speed of evolution gradually increases until rupture occurs. In the simulation, the rupture of a $[\pm45°]_{ns}$ laminate was obtained by introducing a structure instability condition. Under static loading, this condition corresponds to a damage value d=0.5, although this value can be greater under fatigue loading. This result is in line with what was observed experimentally. After a loading process has taken place in a laminate, residual strains can be observed. These strains can be due to the friction occurring between the fibres and the matrix as a result of the damage. To describe them, a kinematical hardening model is under development. This type of anelastic damage behaviour was studied by performing traction tests and 4 point bending tests. The samples used in the bending tests were sandwich structures (a laminate with the ply studied, a honeycomb, and a woven ply laminate at 0°). With this sandwich structure, it is possible to create an almost uniform state of strain in the ply studied, and the [0°] woven plies are assumed to undergo almost no degradation. One of the advantages of the four point bending test is that it makes it possible to prescribe a compression load on the ply under investigation.

3.2 SN curve and residual strength.

The model allows to predict SN curves. A comparison between simulations and tests (Petermann [10]) on $[\pm 45°]_{ns}$ laminates under fatigue loading showed the possibilities of the model (Figure 6). The results of the simulations carried out with different stresses and stress ratios (minimum stress over maximum stress) were in good agreement with the experimental results. The endurance limits were found to vary with respect to the stress ratios in the case of both the model and the experiments.
Finally, it was observed that the residual strength does not decrease with respect to the fatigue loading.

4. High modulus fibres

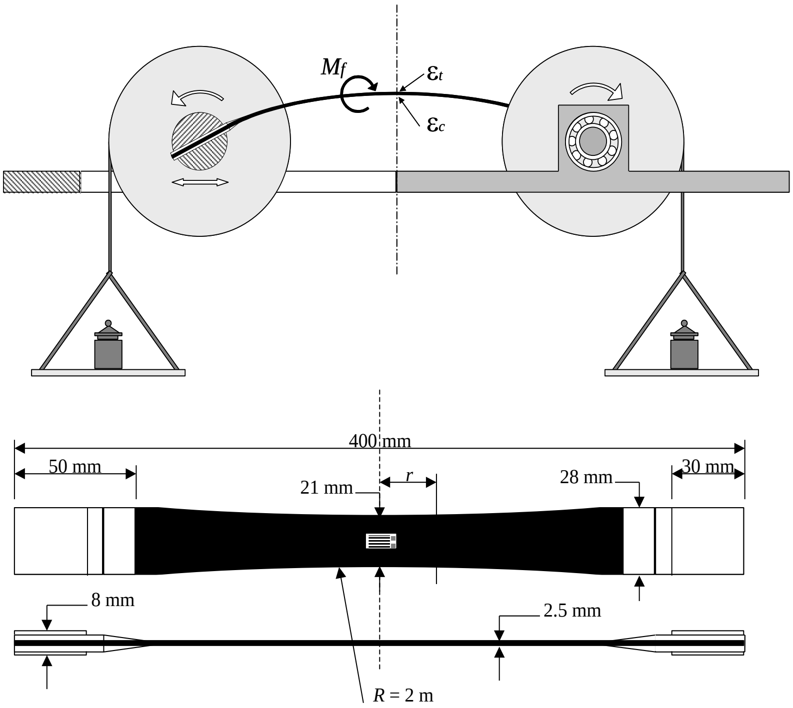


Fig. 7. Diagram and specimen for pure bending test.

The performances of high rotation frequency tubes (such as drive shafts and centrifugal machines) can be improved by using high rigidity carbon fibres. A high effective modulus enables to delay the onset of the vibration modes. The buckling strength under torsion, which is one of the main criteria used in designing hollow laminated composite drive shafts, can also be greatly improved by using these fibres. For these applications, it suffices to model the material in terms of its elastic behaviour according to dynamic and buckling design. It is worth noting that materials of this kind with a very high modulus have a very low breaking strain in the fibre direction, which is even lower than that observed in the transverse direction. In laminates where the plies do not undergo shearing and compression stresses in the fibre direction, elastic criteria can be used to describe the rupture of the laminate under static loading conditions. In the case of compression stresses, the strength of these fibres is very low and their behaviour under high compression conditions therefore needs to be precisely determined.

4.1 Pure bending test

Rupture behaviour under compression loading in the fibre direction is difficult to predict. The standard tests consisting in applying pure compression (Celanese tests) lead to premature rupture because of the edge effects and the buckling of the specimen. Compression tests on $[+60_2,0,-60_2]_s$ laminates [11] allows to decrease the risk of buckling of the specimen. In these compression tests, the calculation of the stresses in the 0° plies is rather complex, however, due to the non-linear behaviour of the 60° plies. Buckling/bending tests [12] and four point bending tests [13] also make it possible to determine the rupture strain under compression loading. However, these buckling/bending tests do not yield any direct information about the stresses in the beam (the inverse calculations are complex, due to the large displacements and rotations involved).

A pure bending test has been proposed (Figure 7) for determining the compression behaviour more precisely. This test, which consists in applying two bending couples at the two free ends and letting one of the ends translate freely, enables to determine the forces at work (constant moment) throughout the sections. In addition, it is possible here to use machined specimens (Figure 7) in order to rule out the occurrence of edge effects at the level of the tabs. The specimens were instrumented on each face with strain gauges. The behaviour of pitch fibres Dialead K63712 was tested. The behaviour is elastic and non-linear (the rigidity decreases, see Figure 8). The rupture mode observed is original. After point A is reached (Figure 8), the strains increased slowly for a constant moment. Indeed, the compressive strain evolved from 0.18% to 0.40%, in parallel, the tensile strain evolved from 0.16% to 0.27%, and the neutral fibre moved towards the tension zone. We think that point A corresponds locally to the top of the compressive stress/strain curve. From this loading, where the maximum compressive stress was reached, the compressive strains increased but the compressive stresses decreased (Figure 9). We suppose that the equilibrium was relatively stable due to viscous phenomena of the matrix.

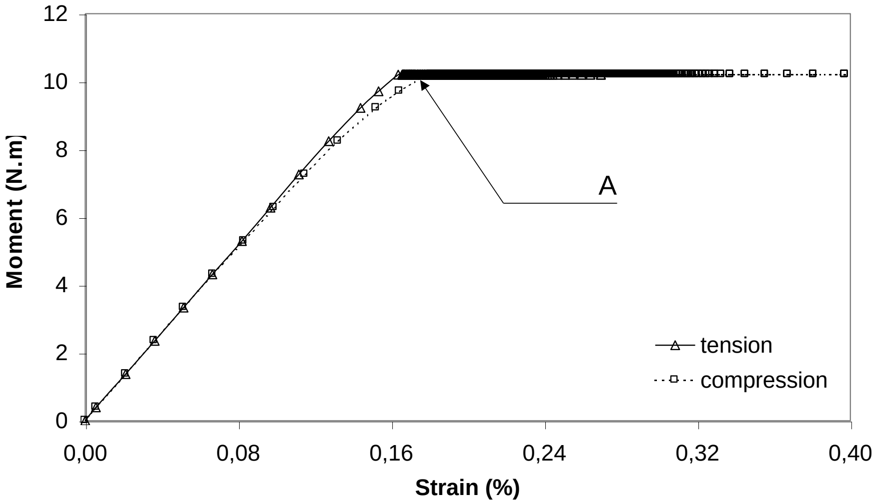


Fig. 8. Pure bending test on unidirectional Dialead K63712

(ε_c: compression face; ε_t: traction face).

4.2 Non-linear behaviour in compression

To model this non-linear behaviour, the rigidity was chosen to decrease under compression (and to increase under tension) depending on the strain. A first model was a linear law between the modulus and the strain [5]. This model was not sufficient here to describe the fast decrease of the compressive modulus. The model used was a power law between the modulus and the strain. The constitutive law, if we add the rigidifying behaviour in tension, is written:

$$\sigma = E\varepsilon + \alpha < -\varepsilon >_+^{n+1} + \beta < \varepsilon >_+^2 \tag{6}$$

The identification of the parameters E and β was realized on tensile tests. The parameters α and n were identified by a minimization method [13] and an additional constraint relative to the rupture associated to the instability process (Figure 9).

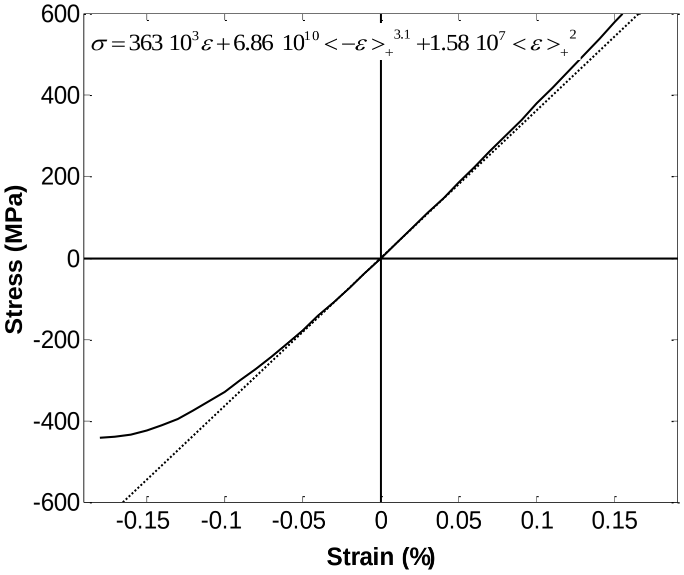


Fig. 9. K63712 constitutive law in the fiber direction.

5. Conclusion

In conclusion, simple rules emerge from the results of this study as far as the technological design of laminated composite structures is concerned:

- *It is recommended to use woven plies to design structures according to strength*;
- *It is recommended to use high-modulus UD plies to design structures according to rigidity*
- *UD plies can be used to design structures according to strength, but only up to failure of the first ply and for structures which are not subject to delamination*

The choice of type of ply (woven or unidirectional plies) does not depend only on mechanical criteria but also on the simplicity of the corresponding modelling procedures.

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