Nonlinear Analysis and Control of an Aircraft in the Neighbourhood of Deep Stall

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Abstract. When an aircraft is locked in a stable equilibrium at high angle-of-attack, we have to do with the so-called deep stall which is a very dangerous situation. Airplanes with T-tail are mainly concerned with this phenomenon since the wake of the main wing flows over the horizontal tail and renders it ineffective but other aircrafts such as fighters can also be affected.

First the phase portrait and bifurcation diagram are determined and characterized (with three equilibria in a deep stall prone configuration). It allows to diagnose the configurations of aircrafts susceptible to deep stall and also to point out the different types of time evolutions. Several techniques are used in order to determine the basin of attraction of the stable equilibrium at high angle-of-attack. They are based on the calculation of the stable manifold of the saddle-point equilibrium at medium angle-of-attack.

Then several ways are explored in order to try to recover from deep stall. They exploits static features (such as curves of pitching moment versus angle-of-attack for full pitch down and full pitch up elevators) or dynamic aspects (excitation of the eigenmodes and improvement of the aerodynamic efficiency of the tail).

Finally, some properties of a deep stall prone aircraft are pointed out and some control tools are also implemented. We try also to apply this mathematical results in a concrete situation by taking into account the captors specificities or by estimating the relevant variables thanks to other available information.

INTRODUCTION

Deep stall occurs when an aircraft is at high angle-of-attack and moreover the horizontal tail which creates the pitching moment is ineffective mainly due to the main wing wake which degrades its aerodynamics. This study deals with the global aircraft behaviour. After modelling the flight dynamics, the core of the analysis focuses itself on the dynamic features such as the characteristics of the eigenmodes, the phase portrait, recovery procedures but some more classical static aspects are also observed such as the multiple longitudinal equilibria (of the pitching moment) and the bifurcation diagram.

MODELLING

A classical (barycentric) model of flight dynamics is taken in this study. Nevertheless the aerodynamics must take into account the effects of deep stall. This is the case for example for the Learjet aircraft model for which data like wind tunnel tests are published in [1] and [2].

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As far as the pitching moment is concerned, it is divided here into two parts. A static part $Cm_{static}(\alpha, \delta_e)$ depending on angle-of-attack α and elevator δ_e. This part is the most important in the sense that it determines the propensity to deep stall. From the practical point of view, an engineer tries to verify that for full pitch up or full pitch down command, no deep stall appears. Another dynamic part renders mostly the damping effect of the tail. Its dependance towards the pitch rate q is linearized. In deep stall, the aerodynamic derivative Cm_q is lower in absolute value since the tail is less efficient. Its mathematical form is identified as a function of α : $Cm_q(\alpha)$.

All in all the aerodynamic coefficient of the pitching moment is

$$Cm(\alpha, q, \delta_e) = Cm_{static}(\alpha, \delta_e) + Cm_q(\alpha) \frac{c_W}{2V} \cdot q \tag{1}$$

with the chord c_W and the speed V.

The other aerodynamic features linked to stall is a lower lift (also an unsteady aerodynamics, not modeled here) and a huge drag which implies amongst other a negative flight-path angle. Indeed the aircraft thrust is no more sufficient to compensate the drag to maintain a level flight in such a way that the aircraft flies down.

After modelling the flight dynamics and aerodynamics, it is possible to analyze the characteristics of a deep stall prone aircraft.

SHORT PERIOD MODE

A first indicator is linked to the characteristics of the short period mode which is a longitudinal (oscillatory) mode and which involves angle-of-attack α and pitch rate q and of low time period.

Indeed the classical linearization of the aircraft model (equations of lift and pitching moment) gives the following formula for the pulsation ω_{spm} and the damping ξ_{spm}.

$$\omega_{spm}^2 = -\frac{V_e^2 \rho c_W S_W}{2I_{YY}} \left[\frac{c_W}{V_e} \left(\frac{\rho S_W V_e}{2m} Cz_\alpha + \frac{T}{mV_e} \right) Cm_q + Cm_\alpha \right] \tag{2}$$

$$2\xi_{spm}\omega_{spm} = \frac{\rho S_W V_e}{2m} Cz_\alpha - \frac{c_W^2 \rho S_W V_e}{2I_{YY}} Cm_q + \frac{T}{mV_e} \cos\alpha_e \tag{3}$$

with the mass m, I_{YY} the moment of inertia about the y-axis, ρ the air density, S_W the main wing surface and the engine thrust T.

In stall, the α-derivative of the lift coefficient Cz_α is lower than in normal operational flight. But in deep stall, the (normalized) q-derivative of the pitching moment coefficient Cm_q is also smaller, since its main contribution comes from the horizontal tail which is under the wake of the main wing and is thus far less effective. As an overall consequence, the damping of the short period mode is smaller at high angle-of-attack than at low angle-of-attack and is far smaller in deep stall. This is a good point to remark in order to develop a sense of danger.

Besides since the damping of the phugoid mode (exchange of altidude/flight-path angle γ and speed V) depends on the lift-to-drag ratio which is very degraded due to the huge drag, generally in stall it is also low. But this remark cannot be used as a discriminant indicator. Moreover as the short period mode is far quicker than the phugoid mode, it will often be assumed that both modes can be decoupled. Thus we will often isolate the behaviour of the short period mode and of the variables (α, q).

PITCHING MOMENT

Amongst others the study of the (static) pitching moment allows to know the longitudinal equilibria since it corresponds to angles-of-attack α and elevator position δ_e for which the pitching moment is zero $Cm_{static}(\alpha, \delta_e) = 0$

(for the static study, the pitch rate $q = 0$). Moreover when the curve of the pitching moment in function of the angle-of-attack decreases (derivative $\frac{\partial Cm}{\partial \alpha} < 0$), the longitudinal equilibrium is statically stable and when the curve increases (derivative $\frac{\partial Cm}{\partial \alpha} > 0$), the longitudinal equilibrium is statically unstable [3].

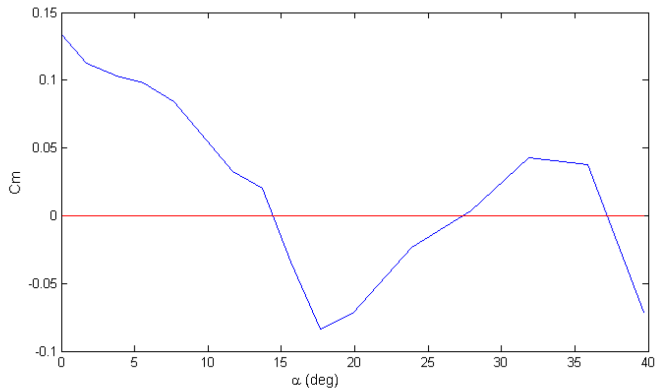

FIGURE 1. Pitching moment coefficient

The figure 1 is typical from a deep stall prone aircraft in the sense that apart from the classical stable equilibrium at low angle-of-attack and the affine (decreasing) pitching moment *Cm*, there are two more equilibria. The equilibrium at medium angle-of-attack is unstable and the one at high angle-of-attack is stable. This last one is linked to the deep stall since it is a stable equilibrium at high angle-of-attack and thus it is a very dangerous situation.

After dealing with the modelling and the static characteristics of deep stall, we will next consider the dynamic aspects. The nonlinear analysis focuses on the classical diagrams of phase portrait (time simulations), bifurcation diagram (curve of equilibria) and then tries to exploit them so as to conclude about the influence of some parameters.

TYPICAL PHASE PORTRAIT

Performing time simulations of the aircraft flight dynamics is the most direct way to study its behaviour since it shows the equilibria, the type and the duration of the involved movements. In the figure 2, the phase portrait shows the three mentioned equilibria. The aircraft can converge to the stable equilibria at low or high angle-of-attack and is repelled from the unstable equilibrium at medium angle-of-attack. This last equilibrium is a so-called saddle point since the Jacobian matrix has two real eigenvalues, one positive and one negative.

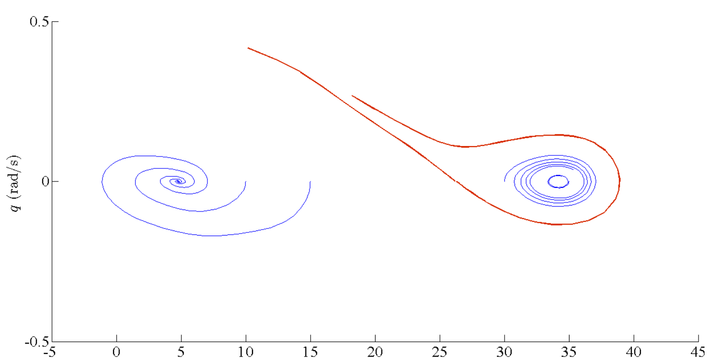

FIGURE 2. Phase portrait

Furthermore it can be noted that the stable manifold of the saddle point is a frontier for the basin of attraction of the equilibrium at high angle-of-attack [4]. This statement comes from the theorem that for sufficient regular planar systems, the trajectories cannot cut themselves [5].

BIFURCATION DIAGRAM

The classical bifurcation diagram of the aircraft can also be drawn with a matlab toolbox like *matcont* [6]. The figure 3 represents the angle-of-attack α at equilibrium in function of the elevator position δ_e and is quite interesting.

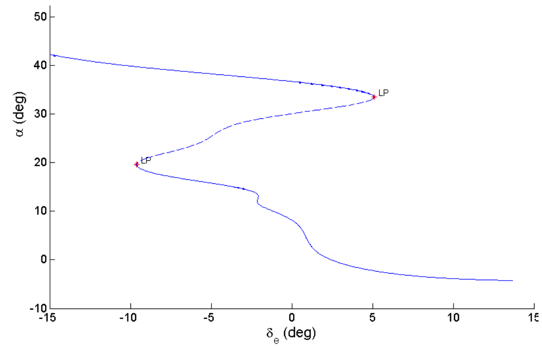

FIGURE 3. Bifurcation diagram

On the one side, there is a range of medium elevator angles for which there are three equilibria. The equilibria at lowest and highest angles-of-attack are stable. They are mostly oscillatory but can be aperiodic stable (the so-called short period mode can be destroyed before becoming unstable at the bifurcation point. Indeed the pair of complex conjugate eigenvalues becomes a pair of negative reals first before one eigenvalue becomes finally positive). As far as the equilibrium at medium angle-of-attack is concerned, it is a saddle point with one real negative eigenvalue and another one real positive. On the other side, for high (positive and negative) elevator angles, there is one unique oscillatory stable equilibrium. The bifurcation points are saddle-nodes since they correspond to a real eigenvalue being zero at this critical parameter. Moreover near a bifurcation point, a *jump* can occur after a little deplacement of the elevator. This sudden event may be dangerous since the pilot does not foresee it and is then locked at high angle-of-attack. Indeed it is not so easy to recover as a phenomenon of hysteresis is visible on the bifurcation diagram (a larger deplacement of the elevator is required so as to recover) and leave the branch of stable equilibria.

After presenting the classical diagrams of dynamical systems linked to this deep stall issue, we will next try to use these elements so as to draw conclusions and to give practical advices to the pilot when flying in the neighbourhood of such a phenomenon.

COMPARISON OF BASINS OF ATTRACTION

The comparison of the sizes of the basins of attraction allows to get an insight about the susceptibility to deep stall. Here the effects of the flight control is assessed and especially concerning the pitch damper. A simple model is here computed with a low and high Cm_q (normalized q-derivative of the pitching moment) in absolute value.

With an activated pitch damper, the q-derivative of the pitching moment is higher (in absolute value), that is to say when there is some pitch rate, the elevator gives rise to a higher pitching moment. In the figure 4, it is visible that the activation of the pitch damper produces a larger basin of attraction. As an advice, the pilot does have to switch off the pitch damper when reaching a deep stall region. It is thus easier to fly back towards the equilibrium at low angle-of-attack.

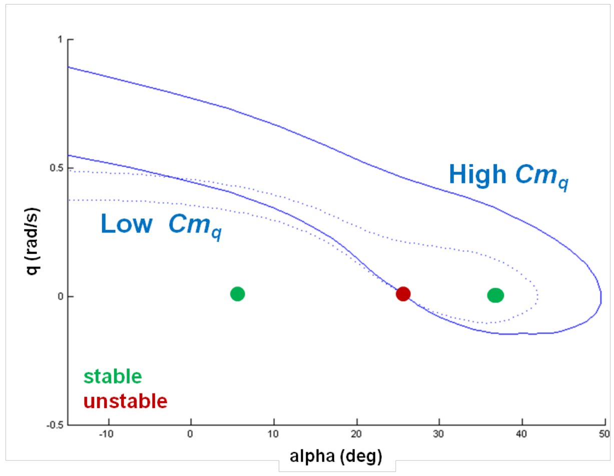

FIGURE 4. Basins of attraction for different Cm_q (with or without pitch damper)

For instance, a passive observation of the aircraft near deep stall was done. Next we will try to become active by finding ways to recover or by adapting the avionics so as to be able to keep on flying with a good level of information in this situation.

RECOVERY

Several recovery procedures are evaluated. Indeed after predicting the deep stall, it is necessary to react adequately if possible. First a static recovery is performed with a classical pitch down command in order to make the airplane recover and the wing aerodynamics to be restored with higher speed. Next a dynamic method based on an oscillating elevator control is made.

Once the Learjet aircraft is stabilized at high AOA, a pitch down command is applied. Depending on the moment this action is applied, the aircraft succeeds in recovering from deep stall or not. The flaps are here down and the center-of-gravity is at 25% of the chord.

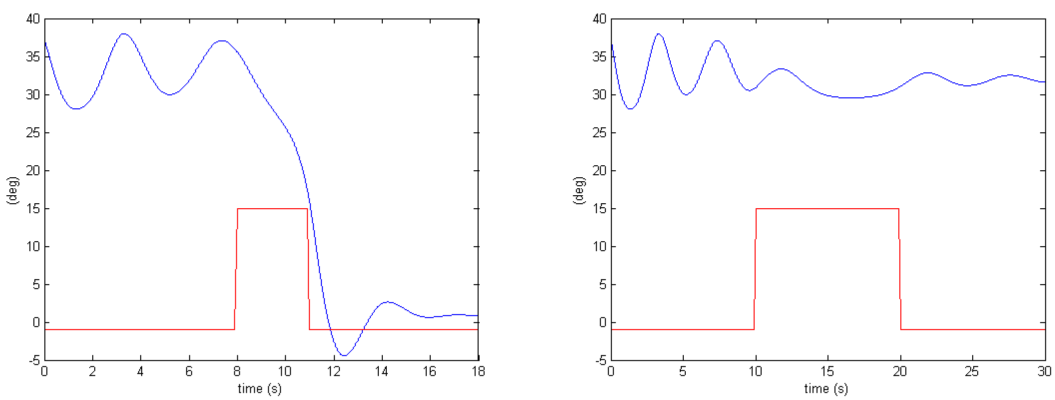

FIGURE 5. Pitch down maneuver for the Learjet aircraft recovery

One question concerns the moment for which a pitch down command is applied. It seems better to do it when the angle-of-attack decreases (with a negative pitch rate moreover) and not when the angle-of-attack increases.

The study performed in the analytical section shows that an abnormally low damping of the short period mode can alert the pilot in advance of a forecoming deep stall. This indication allows to react far in advance and thus to take an appropriate decision as long as it is still possible to do something.

Besides for the F 16 aircraft, a recovery procedure is described in the NASA technical paper [7]. A pitching down moment is created with the speedbrakes and the short period mode is excited so as to create a resonance by an in-phase oscillating action of the pilot.

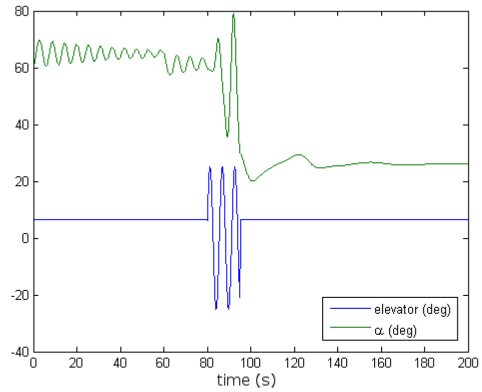

FIGURE 6. Dynamic recovery for the F 16 aircraft

In the figure 6, with an aft-centered aircraft (37.5% of the chord), the described procedure allows well to recover from the deep stall angle-of-attack.

The study is based for instance hion numerical calculations and on theoretical works of modelling and analysis. It is mainly performed with the scientific software *matlab*. But in order to use concretely these knowledges, some practical aspects must be taken into account.

ADAPTED AVIONICS

All this analytical study assumes a good knowledge of the state variables. But the measure of the high angles-of-attack may give problems since it is outside of the range of validity of the usual probes or because some disturbances (vortices) appearing in these conditions may disturb its measure. As a consequence, it is necessary to estimate it with other ways.

FIGURE 7. α-vane employed usually to measure the angle-of-attack.

The first mean consists in choosing probes with higher range of validity like a 5-hole probe (figure 8) instead of the classical α-vane (figure 7). The difference between the pressures allows to determine the angle-of-attack α and the sideslip β [8].

$$C_{P,\alpha} = \frac{P_4 - P_3}{P_{t,ind} - (P_1 + P_2 + P_3 + P_4)} \quad (4)$$

$$C_{P,\beta} = \frac{P_2 - P_1}{P_{t,ind} - (P_1 + P_2 + P_3 + P_4)} \quad (5)$$

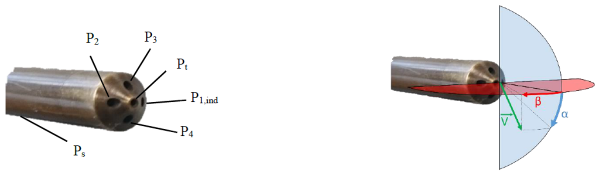

FIGURE 8. 5-hole probes.

The second mean relies on exploiting other informations so as to build estimations of the angle-of-attack or of the variables describing the trajectory. Amongst others GPS, gyroscopes, accelerometers may help estimating such variables even if the AoA probes are out of use. Algorithms such as Kalman filtering or Madgwick method [9] may be implemented in order to use efficiently these captors and to estimate the attitudes. For example, an abnormally huge negative flight-path angle indicates a dangerous situation even if no alert rings elsewhere because the classical probes are not working exceptionally.

CONCLUSION

This study focuses on the deep stall phenomenon. After modelling the aircraft behavior in these conditions, the flight dynamics was analyzed. The typical phase portrait and bifurcation diagram were drawn. Besides an abnormal damping of the short period mode was pointed out in this situation. At the end, some proposals are made in such a way that the pilot recovers from deep stall or that the avionics keeps on working at high angle-of-attack.

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